

DEPARTMENT OF BUILDING ACOUSTICS
LUND INSTITUTE OF TECHNOLOGY

DIFFRACTION BY A SCREEN
ABOVE AN IMPEDANCE BOUNDARY

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SUMMARY

This report is aimed at solving the problem of diffraction by a screen above a locally reacting, infinitely large plane. A general solution is given in terms of an integral equation on the diffracting object. An explicit solution is obtained in the sense of physical optics (Rayleigh solution). Comparisons are made with other solutions by Lindblad and Jonasson. The main disadvantage of these solutions is that their validity is unknown; for sufficiently low screens they are not applicable. The present solution avoids these problems and also extends the knowledge to screens of finite length. To improve the solution numerically a new method of evaluating integrals of two-dimensional Fourier transforms is discussed and applied. This technique may also be used to evaluate the radiated field from a vibrating surface in a plane infinite baffle. In an experimental part a close agreement is found between theory and experiments in model scale indoors as well as outdoors, which also indicates that the method of measuring the ground impedance is useful. The influence of various parameters on the insertion loss is discussed in connection with the model experiments.

INTRODUCTION

General problems of noise reduction by barriers give rise to many interesting theoretical problems, and a recent survey¹ gives an idea of the variety of the problems studied and the methods used. For general aspects on noise reduction by barriers the reader is referred to this survey.

There exists quite extensive knowledge about semi-infinite free field obstacles of different kinds — e.g. screens, wedges, thick barriers, absorbing barriers — whereas the knowledge about finite screens on ground, which is more relevant in practical cases, is rather limited. We know that the excess attenuation over plane grounds of finite impedance may lead to less insertion loss, even negative, than in the corresponding case with hard ground. To the author's knowledge there are only two theoretical studies that have dealt with this problem — by Lindblad² and Jonasson³. For sufficiently low screens both solutions are incorrect and may give rise to large deviations from experimental data. No mathematical restrictions or experimentally found restrictions for the solutions have been given, which makes it difficult to get a general picture of their applicability. Thus there is need for further examination, and although no exact solution shall be given in the present study, it is a contribution to the title problem. This approximate solution avoids the problem with low screens and extends the knowledge to other types of screens, to screens of a finite length. Experimentally the solution shall show to be very useful.

The outline of the report is as follows. In Sec. I we pose the general problem of diffraction or scattering above an impedance boundary. Sec. II repeats the theory of a locally reacting porous layer and its corresponding approximate solution. In Sec. III:B the knowledge from Sec. I and Sec. II is combined and applied to the case with a screen on an impedance boundary. In Sec. III:C an approximate solution, a physical optics solution, is proposed. Sec. III:D repeats and comments other approaches, and in Sec. III:E we shall indicate the relations between the various solutions to this problem. The present solution involves some numerical problems. They are solved in Sec. IV, which shows a new technique of integration (see Sec. IV:C) applicable not only to screens, but also to sound radiation from a vibrating bounded surface in an in-

finite baffle. In Sec. V some numerical aspects of the physical optics solution are reported.

Secs. VI and VII deal with experiments; Sec. VI with extensive model experiments illustrating some aspects of the theory (the influence of ground impedance, receiver height, screen height, screen length, side edge position, angle of incidence and some other types of barriers) and Sec. VII with some outdoor experiments.

I. GENERAL PROBLEM

In this section we shall give the theoretical basis for diffraction or scattering above an impedance boundary.

A. Problem

Introduce a Cartesian co-ordinate system (x, y, z) and consider the velocity potential

$$\psi_D = \psi_D(\vec{r}_R | \vec{r}_S), \quad (1a)$$

where

$$\vec{r}_R = (x_R, y_R, z_R) \quad (1b)$$

and

$$\vec{r}_S = (x_S, y_S, z_S) \quad (1c)$$

outside a body D and in $z > 0$, see Fig. 1. In this domain the velocity

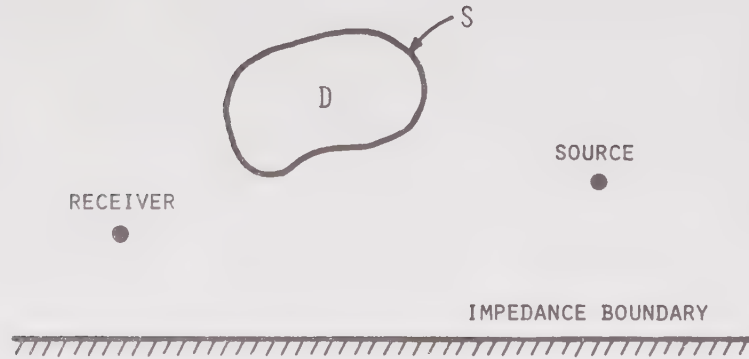


FIGURE 1. The general problem. The source and receiver are located in (x_S, y_S, z_S) and (x_R, y_R, z_R) respectively. The plane $z = 0$ is an impedance boundary with a point admittance v .

potential fulfils the Helmholtz's equation (a common factor $\exp(-i\omega t)$ is suppressed throughout)

$$(\nabla_R^2 + k_0^2)\psi_D = \delta(\vec{r}_R - \vec{r}_S), \quad (2)$$

where k_0 is the (positive) wavenumber and $\vec{\nabla}_R = (\partial/\partial x_R, \partial/\partial y_R, \partial/\partial z_R)$. Thus both the source [located in $\vec{r}_S$ and symbolized by $\delta(\vec{r}_R - \vec{r}_S)$] and the receiver [located in $\vec{r}_R$] are located outside D in $z > 0$.

We do not yet specify the boundary conditions on the surface S of D and at the boundary $z = 0$, but the conditions considered in this context could be summarized as

$$\partial_{nR}\Psi_D + ik_0\nu_0\Psi_D = 0, \quad (3)$$

where

$$\partial_{nR}\Psi_D = (\vec{\nabla}_R \cdot \vec{n}_0)\Psi_D. \quad (4)$$

The point impedance ν_0 , generally complex, may be a function of the position and we allow it to symbolize the perfectly hard ($\partial_{nR}\Psi_D = 0$ and thus $\nu_0 = 0$) and the perfectly soft ($\Psi_D = 0$ and thus $\nu_0^{-1} = 0$) boundaries. $\vec{n}_0$ is the normal to the surfaces of D and of $z = 0$ pointing inward, into the domain studied.

Finally we have to consider Sommerfeld's radiation condition, but as this holds for the incident wave used (See Sec. II), and as this property is not lost when integrating over a bounded domain [see Eq. (8) below], we need not go further into this.

The problem is considerably simplified if we know the solution to the problem with only the boundary $z = 0$ present. The latter solution is known when the impedance of the boundary $z = 0$ is a constant, $\nu_0 = \nu$ and this solution, here called solution of impedance boundary, is denoted

$$\Psi_L = \Psi_L(\vec{r}_R|\vec{r}_S). \quad (5)$$

Some aspects of Ψ_L will be taken up in Sec. II.

B. Integral solution

If we accept an implicit solution in the form of an integral equation we have the well-known relation⁴

$$\Psi_D = \Psi + \{\Psi_D, \Psi\}_{S_0}, \quad (6)$$

where

$$\{\Psi_D, \Psi\}_{S_0} = \iint_{S_0} [\Psi(\vec{r}_R|\vec{r})\partial_n \Psi_D(\vec{r}|\vec{r}_S) - \Psi_D(\vec{r}|\vec{r}_S)\partial_n \Psi(\vec{r}_R|\vec{r})]dS. \quad (7)$$

Integration and differentiation are performed in the (x, y, z) -space. S_0 is the boundary surface (in our case S and $z = 0$) and Ψ is a solution of Eq. (2), usually the free space ("fundamental") solution. In our case it is more convenient to choose $\Psi = \Psi_L$ (i.e. the solution of impedance boundary), which is permissible as Ψ_L has the desired properties in $z > 0$. Then both Ψ_D and Ψ_L satisfy the same boundary condition at $z = 0$ [Eq. (3)], so the integrand of Eq. (7) vanishes on $z = 0$ and Eq. (6) is reduced to an integral over the surface of the diffracting object only:

$$\Psi_D = \Psi_L + \{\Psi_D, \Psi_L\}_S. \quad (8)$$

Now the really difficult problem is to solve the integral equation, i.e. to find Ψ_D on S .

An example of an application of Eq. (8) is scattering from local variations of the surface impedance at $z = 0$. If the impedance varies within an area A on $z = 0$, but otherwise is constant, this problem is reduced to the one of finding the solution to an integral equation [Eq. (8)] on A , see Morse-Ingard⁵.

Before we apply Eq. (8) to the problem of diffraction by a screen above an impedance boundary, as will be done in Sec. III, some comments will be made on the solution to the problem of an impedance boundary, as it is by no means simple to evaluate it.

II. IMPEDANCE BOUNDARY

A. Problem

The problem is defined by the velocity potential

$$\psi_L = \psi_L(\vec{r}_R | \vec{r}_S), \quad (9a)$$

which for $z > 0$ obeys the Helmholtz's equation

$$(\nabla_R^2 + k_0^2)\psi_L = \delta(\vec{r}_R - \vec{r}_S), \quad (9b)$$

subject to the boundary condition

$$(\partial/\partial z_R + ik_0 v)\psi_L = 0 \quad \text{at} \quad z_R = 0+ \quad (9c)$$

and to the radiation condition.

B. Solution

It is fairly simple to obtain a solution by means of the Fourier transform. In its original form the solution is numerically unsatisfying, and different studies often deal with the problem of rewriting it in a suitable form. The numerically most efficient exact solution is the one proposed by Ingard⁶. This solution gives the result in terms of a single integral along a steepest descent contour. To Ingard's solution should be added, for certain receiver-source geometries and certain admittances v , a surface-wave term⁷. Later we shall see that ψ_L is to be integrated over the screen [see e.g. Eq. (23)] and this is a reason to search for a still more efficient solution. Therefore approximations have to be made. To be able to do so we must know the impedance v in the entire frequency range of interest. In a previous study⁸ of which the present one is a continuation, this problem was treated. The starting point is to regard the surface as consisting of a layer. It is shown that, by assuming the refraction index n of the layer to be large, the layer (seen from the medium above it) behaves like a locally reacting surface even for a point source and at grazing incidence. By assuming the layer to be porous we get some knowledge of the frequency dependence of the admittance. In fact it is suggested that⁹

$$v = a(b'/c') \tan(b'c'f/d)/i, \quad (10a)$$

where

$$b' = e^{ib}, \quad (10b)$$

$$c' = [1 + ic(2\pi f)^{-1}]^{1/2}; \quad 0 < \arg(c') < \frac{1}{4}\pi \quad (10c)$$

and where $f = \omega/2\pi$ is the frequency. The real parameters a , b , c and d are unknown but may be estimated by fitting measured and theoretical frequency responses when the source and the receiver are located close to the ground¹⁰. Also, as a result of these assumptions, it is possible to obtain the desired approximate solution¹¹ needed later on in this work.

For the sake of completeness we repeat the approximate solution to the problem of a locally reacting porous layer. We may write this as¹²

$$\psi_L(\vec{r}_R|\vec{r}_S) = \psi_I + \psi_H + \psi_1 + \psi_2, \quad (11)$$

where

$$\psi_I = -(4\pi R_1)^{-1} \exp(ik_0 R_1), \quad (12a)$$

$$\psi_H = -(4\pi R_2)^{-1} \exp(ik_0 R_2) \quad (12b)$$

and

$$\psi_1 = \frac{Ck_0 v}{2\pi^{\frac{1}{2}} B} \exp(ik_0 R_2 \gamma_0) \operatorname{erfc}(A), \quad (12c)$$

where $\operatorname{erfc}(A)$ is the complementary error function. For $|A|^2 < 10$ we use the expansion

$$\operatorname{erfc}(A) = 1 - \frac{2A}{\pi^{\frac{1}{2}}} \sum_{m=0}^{\infty} \frac{(-A^2)^m}{m!(2m+1)}, \quad (13a)$$

and for $|A|^2 \geq 10$ the asymptotical series

$$\operatorname{erfc}(A) \sim \frac{\exp(-A^2)}{A\pi^{\frac{1}{2}}} \sum_{m=0}^{\infty} \frac{(2m)!}{m!(-4A^2)^m}. \quad (13b)$$

Further we have defined

$$\Psi_2 = \begin{cases} \Psi_3 & ; \quad \operatorname{Im}(v) < 0; \operatorname{Re}(\gamma_0) > 1, \\ 0 & ; \quad \text{otherwise,} \end{cases} \quad (14a)$$

and

$$\Psi_3 = k_0 v [\pi^{\frac{1}{2}} B]^{-1} \exp(ik_0 R_2 \gamma_0), \quad (14b)$$

with the constants

$$A = [ik_0 R_2 (\gamma_0 - 1)]^{1/2}, \quad (15a)$$

$$B = [ik_0 R_2 (1 - \gamma_1)]^{1/2}, \quad (15b)$$

$$C = \begin{cases} -1 & ; \quad \operatorname{Im}(v) < 0; \operatorname{Re}(\gamma_0) > 1 \\ 1 & ; \quad \text{otherwise} \end{cases} \quad (15c)$$

$$R_1 = [r_1^2 + (z_S - z_R)^2]^{1/2}, \quad (16a)$$

$$R_2 = [r_1^2 + (z_S + z_R)^2]^{1/2}, \quad (16b)$$

$$r_1 = [(x_S - x_R)^2 + (y_S - y_R)^2]^{1/2}, \quad (17a)$$

$$h = z_S + z_R, \quad (17b)$$

$$\cos(\theta_0) = h/R_2; \quad 0 \leq \theta_0 < \pi/2 \quad (17c)$$

and

$$\gamma_0 = -v \cos(\theta_0) + (1 - v^2)^{1/2} \sin(\theta_0), \quad (18a)$$

$$\gamma_1 = -v \cos(\theta_0) - (1 - v^2)^{1/2} \sin(\theta_0). \quad (18b)$$

All complex roots, $W^{1/2}$, are defined as $-\frac{1}{2}\pi \leq \arg(W^{1/2}) < \frac{1}{2}\pi$. The

conditions for this approximate solution are given by¹³ (apart from the condition $|n|^2 \gg 1$, which makes the boundary locally reacting)

$$k_0 R_2 \gg 1 \tag{19a}$$

and

$$|\gamma_1 - 1| \gg |\gamma_0 - 1|. \tag{19b}$$

III. SCREEN ON IMPEDANCE BOUNDARY

A. Problem

This problem may be considered as a special case of the one in Sec. I, see Fig. 2. The boundary condition at $z = 0$ is that of Eq. (9c) and the point impedance v at $z = 0$ is a constant independent of position.

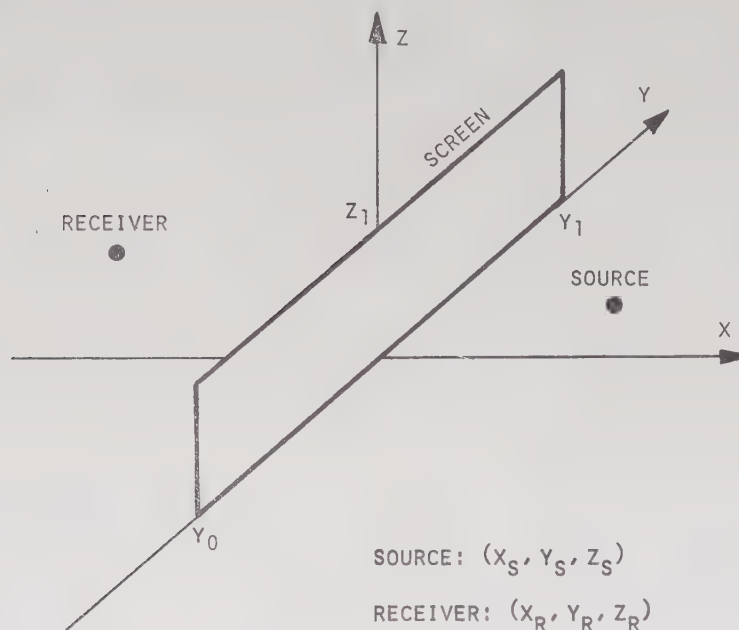


FIGURE 2. Geometry of source (x_S, y_S, z_S) , receiver (x_R, y_R, z_R) and screen. The screen is in the plane $x = 0$: $Y_0 \leq y \leq Y_1$ and $0 \leq z \leq Z_1$. The plane $z = 0$ is an impedance boundary with a point admittance v .

The body D is now a screen in the plane $x = 0$. More specifically the area $Y_0 \leq y \leq Y_1$; $0 \leq z \leq Z_1$, and the boundary condition for the screen is either the perfectly hard or the perfectly soft one. The solutions are denoted $\psi_{\text{Hard}} = \psi_{\text{Hard}}(\vec{r}_R | \vec{r}_S)$ and $\psi_{\text{Soft}} = \psi_{\text{Soft}}(\vec{r}_R | \vec{r}_S)$ respectively. It is pointed out that the condition "perfectly soft boundary" (from now on "perfectly" is understood) should not be confused with "absorbing boundary". Further we assume that the source is located in $x > 0$ (i.e. $x_S > 0$) and that the receiver is located in $x < 0$ (i.e. $x_R < 0$) as in Fig. 2.

B. Solution

By applying the results of Sec. I we find that according to Eq. (8) the solutions may be written as (hard screen):

$$\Psi_{\text{Hard}} = \Psi_L - \iint_S \Psi_{\text{Hard}}(\vec{r}|\vec{r}_S) \partial_n \Psi_L(\vec{r}_R|\vec{r}) dS \quad (20a)$$

and (soft screen)

$$\Psi_{\text{Soft}} = \Psi_L + \iint_S \Psi_L(\vec{r}_R|\vec{r}) \partial_n \Psi_{\text{Soft}}(\vec{r}|\vec{r}_S) dS \quad (20b)$$

We note that once the solutions (Ψ_{Hard} or Ψ_{Soft}) are known on the screen, the solution is known everywhere via Eq. (20). In principle it is possible to obtain a solution by solving Eqs. (20a) or (20b) numerically, but here we shall choose another way and we start by adding some comments on free field screens.

C. Physical optics solution

General remarks on diffraction by plane obstacles in free field and different approaches to obtain solutions may be found in Bouwkamp¹⁴. Some other methods of treating acoustical diffraction and scattering may also be found in the literature¹⁵. We shall not go into all these methods here; the aim of this study is not to treat general problems of screens. We are merely interested in a method leading to an explicit solution. One is the Kirchhoff theory, in which Ψ_D and $\partial_n \Psi_D$ on the shadowed side of the screen (in the sense of optics) are zero and on the illuminated side are given those values they would have if the screen were absent. In connection with Eqs. (20a) and (20b) it is more convenient to use a modification of this theory, based on formulae from Rayleigh. The corresponding solutions are often called "Rayleigh solutions" or solutions in the sense of "physical optics". The postulates are¹⁵

$$\begin{pmatrix} \Psi_D \\ \partial_n \Psi_D \end{pmatrix} = \begin{cases} \begin{pmatrix} 2\Psi \\ 2\partial_n \Psi \end{pmatrix} & \begin{array}{l} \text{on the illuminated} \\ \text{side of the screen} \end{array} \\ \begin{pmatrix} 0 \\ 0 \end{pmatrix} & \begin{array}{l} \text{on the shadowed} \\ \text{side of the screen} \end{array} \end{cases} \quad (21)$$

where the upper case corresponds to a hard screen [Eq. (20a)] and where Ψ is the free field solution. Thus Ψ_D or $\partial_n \Psi_D$ equals twice the incident values. Unfortunately it is not possible to find necessary and sufficient conditions for the validity of this method; it is open to criticism. Nevertheless there have been several fruitful applications of it. A general discussion of this approach may be found in Bouwkamp¹⁶.

We revert to the screen on the impedance boundary and apply the assumptions of "physical optics", and when used we hereafter distinguish it by a superscript "P.O.". The solutions in this sense are given by

$$\Psi_{\text{Hard}}^{\text{P.O.}}(\vec{r}_R | \vec{r}_S) = \Psi_L + \Psi_{\text{Hard}}^{\text{Diff}} \quad (22a)$$

and

$$\Psi_{\text{Soft}}^{\text{P.O.}}(\vec{r}_R | \vec{r}_S) = \Psi_L + \Psi_{\text{Soft}}^{\text{Diff}}, \quad (22b)$$

where $[\partial \Psi_L(\vec{r}_R | \vec{r}) / \partial x = -\partial \Psi_L(\vec{r}_R | \vec{r}) / \partial x_R$ and $\partial \Psi_L(\vec{r} | \vec{r}_S) / \partial x = -\partial \Psi_L(\vec{r} | \vec{r}_S) / \partial x_S]$

$$\Psi_{\text{Hard}}^{\text{Diff}} = 2 \int_0^{Z_1} \int_{Y_0}^{Y_1} \Psi_L(\vec{r}_0 | \vec{r}_S) \partial \Psi_L(\vec{r}_R | \vec{r}_0) / \partial x_R \, dy dz, \quad (23a)$$

$$\Psi_{\text{Soft}}^{\text{Diff}} = -2 \int_0^{Z_1} \int_{Y_0}^{Y_1} \Psi_L(\vec{r}_R | \vec{r}_0) \partial \Psi_L(\vec{r}_0 | \vec{r}_S) / \partial x_S \, dy dz \quad (23b)$$

and

$$\vec{r}_0 = (0, y, z). \quad (24)$$

The solution in the sense of physical optics is exact in two special cases, namely when $Y_1 = -Y_0 = Z_1 = \infty$ [then the assumptions in Eq. (21) hold] and when $Z_1 = 0$ (trivial). This means that for $x_R < 0$ and $x_S > 0$

$$\Psi_L = -2 \int_0^\infty \int_{-\infty}^\infty \Psi_L(\vec{r}_0 | \vec{r}_S) \partial \Psi_L(\vec{r}_R | \vec{r}_0) / \partial x_R \, dy dz \quad (25)$$

and indicates a relation to Babinet's principle¹⁶ for the hard screen

in the sense of physical optics. Eq. (25) suggests another way of writing Eqs. (22a) and (22b). E.g. for a hard screen of infinite length ($Y_1 = -Y_0 = \infty$) we have

$$\Psi_{\text{Hard}}^{\text{P.O.}} = -2 \int_{Z_1}^{\infty} \int_{-\infty}^{\infty} \Psi_L(\vec{r}_0 | \vec{r}_S) \partial \Psi_L(\vec{r}_R | \vec{r}_0) / \partial x_R \, dy dz \quad (26)$$

It might be interesting to note that we must have, according to the reciprocity principle¹⁷

$$\Psi_{\text{Hard}}(\vec{r}_R | \vec{r}_S) = \Psi_{\text{Hard}}(\vec{r}_S | \vec{r}_R), \quad (27)$$

but that the physical optics solution obeys

$$\Psi_{\text{Hard}}^{\text{P.O.}}(\vec{r}_R | \vec{r}_S) = \Psi_{\text{Soft}}^{\text{P.O.}}(\vec{r}_S | \vec{r}_R) \quad (28)$$

as now the opposite side of the screen is illuminated and as Ψ_L fulfils the reciprocity principle. Obviously the physical optics solution is incapable of separating the effect of a perfectly soft and a perfectly hard screen — if it does, it also violates the reciprocity condition. The calculations in the experimental part of this paper have shown that the difference between the soft and the hard screen seldom exceeds 0.3 dB in the cases studied (the only exception was for oblique incidence on the screen; 60 deg., see Sec. VI:G). Once $\Psi_L(\vec{r}_R | \vec{r}_0)$ and $\Psi_L(\vec{r}_0 | \vec{r}_S)$ [in Eqs. (23a) and (23b)] have been calculated, one may calculate (see Sec. IV:B) $\partial \Psi_L(\vec{r}_R | \vec{r}_0) / \partial x_R$ and $\partial \Psi_L(\vec{r}_0 | \vec{r}_S) / \partial x_S$ with little effort. Therefore, it is suggested that both $\Psi_{\text{Hard}}^{\text{P.O.}}$ and $\Psi_{\text{Soft}}^{\text{P.O.}}$ are calculated and compared. If they differ too much the solution should be used with care.

In this connection there is an important quantity, the insertion loss; the decrease in the level when the screen is inserted. It is defined as

$$IL_{\text{Hard}}^{\text{P.O.}} = 20 \times \log[|\Psi_L / \Psi_{\text{Hard}}^{\text{P.O.}}|] \quad (29a)$$

and

$$IL_{\text{Soft}}^{\text{P.O.}} = 20 \times \log[|\Psi_L / \Psi_{\text{Soft}}^{\text{P.O.}}|] \quad (29b)$$

for the hard and soft screens respectively.

Again it is pointed out that no necessary and sufficient conditions can be given under which the approximate solution is valid. It is also most unfortunate that we have no exact solution to compare it with. One fortunate property of Eqs. (23a) and (23b), though, is that they always obey the boundary condition at $z = 0$, which means that for zero height screen the solution is exact and one might well think that as long as reflection is the dominating phenomenon, the solution works well. This property is also important for outdoor propagation, as we know that at high frequencies, say 1000 Hz, the insertion loss of the screen is often counteracted by the strong attenuation of the ground.

A more accurate solution is desirable and this is one side of the problem. The other side is that although the physical optics solution formally is simple, the integral over the screen is by no means simple to calculate. Ψ_L is in its original expression a two-dimensional Fourier transform and although much more numerically efficient approximate solutions exist, see Sec. II, there is need for a fast digital computer for its evaluation. At the present stage Eq. (23) seems reasonable in view of the calculation costs, at least after having made the analysis in Sec. IV.

The only way left to test its validity is by making experiments. This is theoretically unsatisfactory, but at least it gives us an idea of its practical applicability. For the cases studied and reported in the experimental part of this paper, Secs. VI and VII, there seems to be quite a good agreement with measurements. To remind us that the validity of Eqs. (23a) and (23b) is unknown and that this has to be proved by other means than theoretical ones, we refer to the solution by "physical optics solution" throughout this report and use the superscript P.O.

D. Other approaches

The problem of sound propagation above an impedance boundary with a screen has previously been treated by Lindblad² and Jonasson³. Both studies contain mathematical models and measurements. Apart from the geometry one essential parameter has to be estimated, the admittance v . Lindblad used a modified impedance tube and Jonasson applied methods;

of fitting measured and theoretical curves, both methods at a given frequency. The solutions are repeated here in order to illustrate the relations between the different approaches, which will be done in the next subsection. As the solution in Sec. III:C, these solutions do not differ for the hard and the soft screen cases.

Lindblad's basic ideas behind his model are repeated here. The pressure on the receiving side of the screen could be expressed as an integral over elementary sources as in Eq. (26). The velocity above the screen is thought to be proportional to the pressure. The solution is built up of four different types of waves -- direct and reflected -- on each side of the screen. The reflection coefficients are simply those of a plane wave, here denoted R_s and R_r , on the source and the receiver side of the screen respectively. Then the pressure at the receiver may be written as¹⁸

$$p_L = C \int_{Z_1}^{\infty} [1 + R_s f_s][1 + R_r f_r] f_T dz \quad (30)$$

where f_s , f_r and f_T are functions taking the different paths of the sound into account and C is a proportional constant.

Jonasson's approach is different from Lindblad's. The problem with the screen on the ground is thought to be similar to the one with a semi-infinite screen placed between two sources (the real source and an image source) and two receivers (the real receiver and an image receiver). The image source and the image receiver are obtained from the exact solution of the impedance boundary; the spherical "reflection coefficient" Q from Ingard¹⁹ is used. After some analysis it is shown, in the cases regarded as practically important, that the diffraction and the reflection may be separated in the sense that we need not calculate the diffraction of each plane wave component. Instead we calculate the diffraction from a point source of unit strength and multiply by Q for each reflection at the surface $z = 0$. The pressure at the receiving point is a sum²⁰

$$p_J = p_{11} + p_{12} + p_{21} + p_{22}, \quad (31a)$$

where $(k, l = 1$ corresponds to real source and receiver and $k, l = 2$

to imagined ones)

$$p_{kL} = \frac{\exp[i(k_0 R'_{kL} - \pi/4)]}{k_0 (R_1)_{kL} \pi^{1/2}} \text{Fr}(x_{kL}) Q_{kL} \quad (31b)$$

$$\text{Fr}(s) = \int_s^{\infty} \exp(it^2) dt \quad (31c)$$

$$x_{kL} = \{k_0 [(R_1)_{kL} - R'_{kL}]\}^{1/2} \quad (31d)$$

$$Q_{11} = 1; \quad Q_{22} = Q_{12} Q_{21} \quad (31e)$$

$(R_1)_{kL}$ is the distance (image) source - (image) receiver via the top of the screen, and R'_{kL} is the corresponding distance the shortest way from the source to the receiver. Q_{12} and Q_{21} are the spherical reflection coefficients from Ingard¹⁹ determined by the angles of reflection (defined by the image source or image receiver and the top of the screen) and the distances $(R_1)_{12}$ and $(R_1)_{21}$.

Both solutions are easy to calculate and can be used without extensive calculation costs. Therefore an evident application is "design charts" for traffic noise. Another property in common is that the screen is of infinite length. Neither of the solutions approaches the exact one when the height of the screen goes to zero. For this interesting case the solution by Lindblad is far better than using plane wave reflection only, but it may differ considerably from the exact solution²¹.

Jonasson's solution, which has the correct spherical reflection coefficients, is in general difficult to penetrate for the zero height screen, but as an example we assume that the source and receiver are located on the ground, i.e. that $z_S = z_R = 0$ and $Z_1 = 0$. Further we assume that $y_S = y_R = 0$. According to Eq. (31) the pressure becomes

$$p_J = \exp(ik_0 R') (1 + Q)^2 (2k_0 R')^{-1}, \quad (32a)$$

where

$$R' = x_S - x_R, \quad (32b)$$

whereas the exact solution to this problem is

$$p_E = \exp(ik_0 R')(1 + Q)(k_0 R')^{-1}. \quad (33)$$

Here Q is the spherical reflection coefficient for the distance R' between source and receiver when they are located on the ground. For a perfectly hard boundary ($Q = 1$) or a perfectly soft one ($Q = -1$), p_J is obviously equal to p_E as it should be, but for an impedance boundary this is not the case. As an example, impose the conditions $k_0 R' \gg 1$; $k_0 R' |v|^2 \gg 1$ and choose v so that surface wave contribution is no part of the solution. Then, according to Wenzel²², we have

$$1 + Q = 2i(v^2 k_0 R')^{-1}, \quad (34)$$

and we see that p_J and p_E differ considerably, as $|p_E/p_J| = k_0 R' |v|^2 \gg 1$.

This shows the difficulties in applying the theory of semi-infinite screens to problems of finite screens; the critical point is the formulation of the problem; to what extent it is allowed. How high the screen must be to behave like a semi-infinite screen in the entire frequency range is not known; neither mathematically nor experimentally found limitations have been given.

E. Comparisons with other approaches

The solutions by Lindblad and Jonasson have a connection to the present theory and to each other. For the sake of completeness, and as it has not been pointed out earlier, we shall show how. We start by recalling Eq. (26), and for the sake of simplicity we put $y_R = y_S = 0$ and $z_R = z_S = 0$. We also assume that the screen is low in the sense that $Z_1 |x_R|^{-1}$ and $Z_1 |x_S|^{-1}$ are small compared to unity, which is usually the case. Then, if $|x_R k_0|$ is much larger than unity, the normal velocity is approximately proportional to the pressure on the screen, as assumed a priori by Lindblad and Jonasson. This may be seen in Sec. IV:B. In fact we have [Eqs. (47a)-(47d)] for small y and z

$$\partial \Psi_L / \partial x_R \cong ik_0 x_R \Psi_L / |x_R| = -ik_0 \Psi_L. \quad (35)$$

Also by Taylor expansion (for small y and z and for $k_0 \ll 8x_S^3/Z_1^4$) roughly

$$\psi_L(\vec{r}_0|\vec{r}_S) \cong \psi_S \exp[ik_0 \frac{1}{2}(y^2 + z^2)/x_S], \quad (36)$$

$$\psi_S = \psi_L(\vec{r}_{Z_1}|\vec{r}_S) \times \exp[ik_0(x_S - R_S)], \quad (37)$$

where $R_S = (x_S^2 + Z_1^2)^{1/2}$ and $\vec{r}_{Z_1} = (0, 0, Z_1)$.

A similar relation holds for $\psi_L(\vec{r}_R|\vec{r}_0)$ with $R_R = (x_R^2 + Z_1^2)^{1/2}$. Introduce the approximations Eqs. (35) and (36) into Eq. (26) and integrate with respect to y and z . One finds that

$$\psi_{\text{Hard}} \cong \psi_T \pi^{-\frac{1}{2}} \exp(-i\pi/4) \text{Fr}(z')(1 + Q_R)(1 + Q_S) \quad (38)$$

where (note that $x_R < 0$)

$$z' = Z_1[k_0(x_S^{-1} - x_R^{-1})/2]^{1/2}, \quad (39a)$$

$$\psi_T = -[4\pi R']^{-1} \exp(ik_0 R') \quad (39b)$$

$$Q_S = -\psi_S \exp(-ik_0 R_S) 4\pi R_S - 1 \quad (39c)$$

and similarly for Q_R . R' is defined in Eq. (32b) and $\text{Fr}(\)$ in Eq. (31c).

As $z' \cong x_{kZ}$ this result differs from Jonasson's, Eq. (31), essentially in the definition of Q_R and Q_S (corresponding to Q_{12} and Q_{21} , which are defined for the same angle of incidence as Q_R and Q_S but both for the distance $R_R + R_S$ instead of R_R and R_S as with Q_R and Q_S). In most practical cases this has a minor influence on the results. For a zero height screen, $Z_1 = 0$, we face the same trouble as with Jonasson's solution [see Eqs. (32a) and (33)] and in view of the analysis leading to Eq. (38) we may interpret this as a consequence of the high attenuation near the ground [see Eq. (34)], which makes even large z -values [for which the expansion Eq. (36) is not valid] contribute to

the integral Eq. (26).

We may note that for large numerical distances A [see Eq. (15a)] the spherical wave reflection coefficient Q tends to the plane wave reflection coefficient. Then we have essentially arrived at Lindblad's solution, although Lindblad did not separate reflection and diffraction, but instead integrated numerically [see Eq. (30)]. Another difference between Lindblad's and Jonasson's approaches is that Lindblad for the case without a screen simply puts $Z_1 = 0$ in Eq. (30), whereas Jonasson uses the exact solution by Ingard.

The essential difference between the present solution, Sec. III:C, compared to Lindblad's and Jonasson's, is that no further approximations [like Eqs. (35) and (36)] except the one of physical optics [Eq. (21)] are made, and that integrals on the screen rather than integrals above the screen are studied [cf. Eqs. (22a) and (26)]. Thus the present solution ought to be a step forward in accuracy and also an extension to cases not covered by other models, e.g. a screen of finite length. Although approximations similar to Eqs. (35) and (36) could be made to the integral over the screen [Eqs. (23a) or (23b)] it will be possible to calculate the integrals in such an efficient way (without introducing more approximations, the consequences of which are difficult to penetrate) that we restrain ourselves from doing so. The price for this is higher computation costs. First we will make an analysis of the diffracted field in order to obtain a numerically more suitable solution.

IV. ANALYSIS OF DIFFRACTED FIELD

When we evaluate the integrals of Eqs. (23a) or (23b) we are bound to use some approximate solution of the integrand — i.e. of Ψ_L — otherwise we shall have to treat rather difficult integrals of four variables. The approximate solution of Ψ_L to be used is the one from Sec. II:B. It is valid under certain conditions, see Eqs. (19a) and (19b). These conditions may in fact be used to simplify not only the evaluation of Ψ_L , but also the evaluation of the integrals over the screen. This is fortunate; once having made approximations, we want to use them as much as possible in the analysis of the diffracted field in order to get as numerically efficient solutions as possible.

There are two specific points where approximations can be made without introducing new conditions; when we evaluate $\partial\Psi_L/\partial x_R$ and when we integrate with respect to y and z in Eqs. (23a) and (23b). It is the aim of this section to show how. The analysis is made in the subsections A and C, and the complete results are presented in separate subsections (B and D). For the reader interested in the results only, it is sufficient to consult the last-mentioned subsections.

Radiation from a vibrating surface in a rigid panel is closely related to the diffraction theory²³ in the sense of physical optics. One term of the series Eq. (65c) below is very similar to the far-field from a vibrating rectangular plate²⁴. By dividing a vibrating rectangular plate into segments, as we shall do with the screens, it should be possible to evaluate the field "near" a vibrating plate too, in terms of series similar to Eq. (65c).

A. $\partial\Psi_L/\partial x_R$ — analysis

The exact solution of Ψ_L may be written as a double integral or as a single integral along a steepest descent contour. However, approximate solutions have to be used when evaluating Eqs. (23a) and (23b). This makes numerical differentiation unnecessary. Below we shall show how.

The exact solution of Ψ_L (i.e. of the problem of impedance boundary) may be written as²⁵

$$\Psi_L = \Psi_I + \Psi_H + \Psi_{LR}. \quad (40)$$

Ψ_I and Ψ_H are given by Eq. (12),

$$\Psi_{LR} = - \frac{ik_0 v}{(2\pi)^2} \iint_{\Omega} \frac{\exp[ik_0(r_1 \alpha + h\gamma)]}{v + \gamma} d\phi d\gamma, \quad (41a)$$

and

$$\alpha = (1 - \gamma^2)^{1/2} \cos(\phi). \quad (41b)$$

r_1 and h are given by Eq. (17) and complex roots $W^{1/2}$ are defined as $-\pi/2 \leq \arg(W^{1/2}) < \pi/2$. The domain of integration, Ω , is $0 \leq \phi \leq 2\pi$ and $\gamma \in L_1$, see Fig. 3. The contour L_1 has been chosen to avoid trouble with the pole at $\gamma = -v$ in the subsequent analysis.

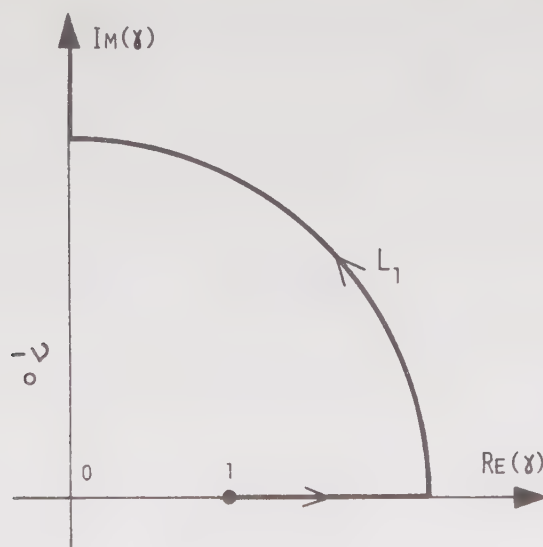


FIGURE 3. Contour L_1 .

$\partial\Psi_I/\partial x_R$ and $\partial\Psi_H/\partial x_R$ may be written down directly and are omitted in the following discussion. The problem of rewriting $\partial\Psi_{LR}/\partial x_R$ remains. If we differentiate with respect to x_R under the integral sign of Eq. (41a) we find that

$$\partial\Psi_{LR}/\partial x_R = C_1 \iint_{\Omega} \frac{\alpha \exp[ik_0(r_1 \alpha + h\gamma)]}{v + \gamma} d\phi d\gamma, \quad (42a)$$

where

$$C_1 = k_0^2 v(x_R - x_S) / [(2\pi)^2 r_1]. \quad (42b)$$

Introduce new variables of integration, ϕ' and γ' , defined by

$$\alpha' = \alpha \cos(\theta_0) - \gamma \sin(\theta_0), \quad (43a)$$

$$\gamma' = \gamma \cos(\theta_0) + \alpha \sin(\theta_0), \quad (43b)$$

and with $\cos(\theta_0)$ given by Eq. (17) and α' defined similarly to α in Eq. (41b). By applying Cauchy's integral theorem of two complex variables to the integral of Eq. (42a), as suggested by Weyl²⁶, it is possible to show that the domain of integration (Ω') for γ' and ϕ' is the same as for γ and ϕ , and that the pole at $\gamma = -v$ is of no trouble²⁷. Hence we have

$$\partial \Psi_{LR} / \partial x_R = C_1 \iint_{\Omega'} \frac{\alpha \exp(ik_0 R_2 \gamma')}{v + \gamma} d\phi' d\gamma', \quad (44a)$$

where R_2 is given by Eq. (16b) and

$$\alpha = \alpha' \cos(\theta_0) + \gamma' \sin(\theta_0), \quad (44b)$$

$$\gamma = \gamma' \cos(\theta_0) - \alpha' \sin(\theta_0). \quad (44c)$$

This integral may be simplified if, keeping γ' fixed, we integrate with respect to ϕ' . After some algebra it is found that²⁷

$$\partial \Psi_{LR} / \partial x_R = C_2 \int_{L_1} \exp(ik_0 R_2 \gamma') \frac{\gamma'}{W^{1/2}} d\gamma' + C_3 (\Psi_{LR} + 2\Psi_H) \quad (45a)$$

where γ_0 and γ_1 are given by Eq. (18) and

$$W = (\gamma' - \gamma_0)(\gamma' - \gamma_1), \quad (45b)$$

$$C_2 = C_1 2\pi / \sin(\theta_0) \quad (45c)$$

and

$$C_3 = C_1(2\pi)^2 \cot(\theta_0)i/k_0. \quad (45d)$$

Introduce the contour L_2 , see Fig. 4, in the integral of Eq. (45a).

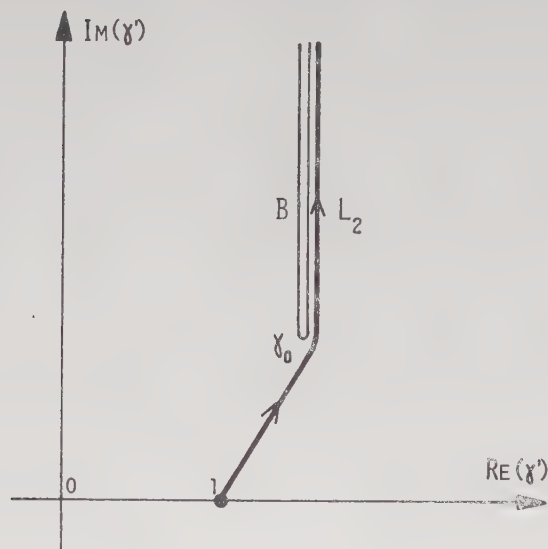


FIGURE 4. Contour L_2 and branch line B of $W^{1/2}$, when $\text{Im}(\nu) < 0$ and $\text{Re}(\gamma_0) > 1$. Otherwise L_2 is defined as $\gamma' = 1 + it$, $t \geq 0$

We now see that an essential contribution originates from $\gamma' = 1$, and in cases such as in Fig. 4 also from $\gamma' = \gamma_0$. The approximate solution to be used is restricted by the two conditions Eq. (19a) and (19b), the latter of which suggests that $\gamma_0 \cong 1$, i.e. that the essential contribution always originates from $\gamma' = 1$. More specifically (compare with the discussion of the approximate solution in reference 8, Sec. VI) the branch line B may be part of $\text{Re}(\gamma') > 1$ when the source and the receiver are located close to the boundary and when $\text{Im}(\nu)$ is smaller than $-\text{Re}(\nu) [1 + \text{Re}^2(\nu)]^{-1/2}$. This happens at low frequencies, where $|\nu|$ is small [see Eqs. (10a)-(10c)], which makes it possible to write $(1 - \nu^2)^{1/2} \cong 1 - \nu^2/2$. This means that $\gamma_0 \cong 1$ at low frequencies and at low source/receiver heights. At high frequencies or high source/receiver heights $\text{Re}(\gamma_0)$ is usually smaller than unity. The branch line and hence the condition Eq. (19b) play a minor part, which indicates that it is possible to put $\gamma' = 1$ in the numerator of Eq. (45a) in the entire interesting frequency range.

Thus, without introducing any more restrictions to our approximate

solution of Ψ_L , we may assume that

$$\exp(ik_0 R_2 \gamma') \gamma' / W^{1/2} \cong \exp(ik_0 R_2 \gamma') / W^{1/2}, \quad (46)$$

which makes the first term of Eq. (45a) equal to $\Psi_{LR} C_3 [v \cos(\theta_0)]^{-1}$.

B. $\partial \Psi_L / \partial x_R$ - result

Under the conditions given by Eqs. (19a) and (19b) the result of the preceding subsection may be written as (we use the notation from Sec. II:B)

$$\partial \Psi_L(\vec{r}_R | \vec{r}_S) / \partial x_R \cong \partial \Psi_I / \partial x_R + \partial \Psi_H / \partial x_R + G[\Psi_{LR} + \cos(\theta_0) v (\Psi_{LR} + 2\Psi_H)], \quad (47a)$$

where

$$\partial \Psi_I / \partial x_R = (x_R - x_S)(ik_0 R_1 - 1)\Psi_I / R_1^2, \quad (47b)$$

$$\partial \Psi_H / \partial x_R = (x_R - x_S)(ik_0 R_2 - 1)\Psi_H / R_2^2 \quad (47c)$$

and

$$G = ik_0(x_R - x_S)[R_2 \sin^2(\theta_0)]^{-1}. \quad (47d)$$

Together with the approximate solution of Ψ_L [Eqs. (10)-(19)], Eqs. (47a)-(47d) constitute the desired approximate solution of $\partial \Psi_L / \partial x_R$ and will be used when evaluating the integrals in Eqs. (23a) and (23b).

When $\theta_0 \rightarrow 0$ we note that $G \rightarrow \infty$. In the exact expression, see Eq. (45a), this is compensated by other terms in order to make $\partial \Psi_{LR} / \partial x_R$ bounded. Due to the approximation Eq. (46) this is not the case with Eq. (47a), but it has no importance for our calculations, as θ_0 is always near $\pi/2$.

Finally, for the cases studied it seems as if the approximate solution of $\partial \Psi_L / \partial x_R$ compared to numerical differentiation of the exact one is as

good as the approximate solution of Ψ_L compared to the exact one²⁸.

C. The integral - analysis

At this point there are no technical difficulties in evaluating the effect of a screen in the sense of the present solution. The integrals of Eqs. (23a) and (23b) may be calculated with a fast digital computer. Nevertheless it was found that if somehow the calculation of the integrals could be made faster, the practical value of the solution would increase. In this subsection we shall show that it is possible to do so. Once the analysis is made, it is simple to make the necessary alterations in the solution. As a by-product we will find a coarse estimate of the error of the integral over the screen, which simplifies the calculation procedure. The analysis is made in the following way: The rectangular screen is divided into rectangular segments, see Fig. 5.

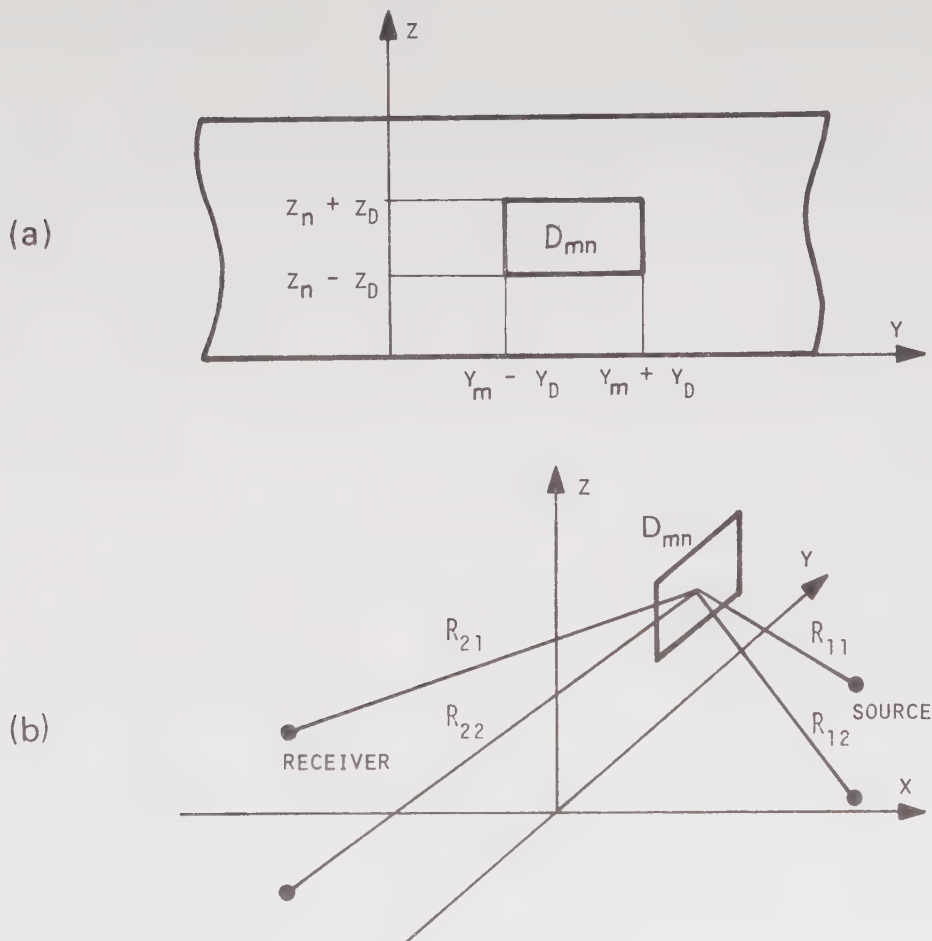


FIGURE 5. Segments D_{mn} . For each segment the integral Eq. (50b) is evaluated and expressed in terms of known functions; Eq. (65c).

It is shown that if these segments are sufficiently small, it is possible to evaluate the integral over the segments by integrating with respect to y and z under the integral sign of the exact solution of Ψ_L . The numerical gain well justifies the somewhat complicated analysis. (Another way of making this analysis is outlined at the end of this subsection.)

We confine ourselves to the case of a hard screen, as the soft screen may be treated in a similar way. We divide the integrand of Eq. (23a) into four parts by defining

$$\Psi_L(\vec{r}_0|\vec{r}_S) = \Psi_{11} + \Psi_{12} \quad (48a)$$

and

$$\Psi_L(\vec{r}_R|\vec{r}_0) = \Psi_{21} + \Psi_{22}. \quad (48b)$$

Ψ_{11} and Ψ_{21} refer to the direct fields and Ψ_{12} and Ψ_{22} are the reflected parts of these fields. Mathematically we have defined, see Eqs. (11) and (12),

$$\Psi_{11}(\vec{r}_0|\vec{r}_S) = \Psi_I(\vec{r}_0|\vec{r}_S), \quad (49a)$$

$$\Psi_{21}(\vec{r}_R|\vec{r}_0) = \Psi_I(\vec{r}_R|\vec{r}_0) \quad (49b)$$

and

$$\Psi_{12}(\vec{r}_0|\vec{r}_S) = \Psi_L(\vec{r}_0|\vec{r}_S) - \Psi_{11}, \quad (49c)$$

$$\Psi_{22}(\vec{r}_R|\vec{r}_0) = \Psi_L(\vec{r}_R|\vec{r}_0) - \Psi_{21}. \quad (49d)$$

We now separate the integral $\Psi_{\text{Hard}}^{\text{Diff}}$ [Eq. (23a)] into four parts as

$$\Psi_{\text{Hard}}^{\text{Diff}} = \Psi_{11}^{\text{Diff}} + \Psi_{12}^{\text{Diff}} + \Psi_{21}^{\text{Diff}} + \Psi_{22}^{\text{Diff}} \quad (50a)$$

where the terms $\Psi_{k\ell}^{\text{Diff}}$ are built up of different combinations of direct and reflected waves as ($k = 1, 2$; $\ell = 1, 2$)

$$\Psi_{k\mathcal{L}}^{\text{Diff}} = 2 \int_0^{Z_1} \int_{Y_0}^{Y_1} \Psi_{1k} \partial \Psi_{2\mathcal{L}} / \partial x_R dy dz. \quad (50b)$$

It is not possible to perform analytical integration of Eq. (50b) with respect to y and z . At this stage it is tempting to integrate the approximate solutions of Ψ_L numerically by approximating Eq. (50b) with a suitable sum. However, a much more powerful approach is possible if instead we revert to the integral expressions of Ψ_L . These expressions can easily be found from the equality

$$|\vec{r}|^{-1} \exp(ik_0 |\vec{r}|) = -ik_0 (2\pi)^{-1} \iint_{\Omega_{\mathcal{L}}} E_{\mathcal{L}}[\vec{r}] d\phi_{\mathcal{L}} d\gamma_{\mathcal{L}} \quad (51)$$

where, as before, the domain of integration, $\Omega_{\mathcal{L}}$, is $0 \leq \phi_{\mathcal{L}} \leq 2\pi$ and $\gamma_{\mathcal{L}} \in L_1$, $\mathcal{L} = 1, 2$ and

$$E_{\mathcal{L}}[\vec{r}] = \exp[i\vec{k} \cdot (x, y, |z|)], \quad (52a)$$

$$\vec{k} = k_0(\alpha_{\mathcal{L}}, \beta_{\mathcal{L}}, \gamma_{\mathcal{L}}), \quad (52b)$$

$$\alpha_{\mathcal{L}} = (1 - \gamma_{\mathcal{L}}^2)^{1/2} \cos(\phi_{\mathcal{L}}), \quad (52c)$$

$$\beta_{\mathcal{L}} = (1 - \gamma_{\mathcal{L}}^2)^{1/2} \sin(\phi_{\mathcal{L}}). \quad (52d)$$

Complex roots $W^{1/2}$ are defined as $-\pi/2 \leq \arg(W^{1/2}) < \pi/2$. As in subsection IV:A, the contour L_1 , Fig. 3, has been introduced to avoid trouble with the poles in the subsequent analysis. It is possible to express all $\Psi_{k\mathcal{L}}$ in terms of similar functions. In fact we have²⁹

$$\Psi_{2\mathcal{L}}(\vec{r}_R | \vec{r}_0) = \frac{ik_0}{8\pi^2} \iint_{\Omega_2} E_2[\vec{r}_0 - \vec{r}_R^{(\mathcal{L})}] A_{2\mathcal{L}} d\phi_2 d\gamma_2, \quad (53a)$$

where the superscript takes care of reflection by

$$\vec{r}_R^{(\mathcal{L})} = \begin{cases} (x, y, z) & ; \mathcal{L} = 1 \\ (x, y, -z) & ; \mathcal{L} = 2 \end{cases} \quad (53b)$$

and where the reflection coefficients are

$$A_{k\ell} = \begin{cases} 1 ; & \ell = 1, \\ \frac{\gamma_k - v}{\gamma_k + v} ; & \ell = 2. \end{cases} \quad (53c)$$

$\vec{r}_0$ is given by Eq. (24). A similar expression for $\psi_{1k}(\vec{r}_0|\vec{r}_S)$ holds. In view of Eqs. (50b) and (53a) we may write

$$\psi_{k\ell}^{\text{Diff}} = C \int_0^{Z_1} \int_{Y_0}^{Y_1} \iint_{\Omega_2} \iint_{\Omega_1} A_{1k} A_{2\ell} F_0 \alpha_2 d\Omega_1 d\Omega_2 dy dz, \quad (54a)$$

where

$$C = -\frac{1}{2} (ik_0)^3 (2\pi)^{-4} \quad (54b)$$

and

$$F_0 = E_1[\vec{r}_0 - \vec{r}_S^{(k)}] E_2[\vec{r}_0 - \vec{r}_R^{(\ell)}]. \quad (54c)$$

It is possible to change the order of integration in Eq. (54a) and directly integrate over the entire screen. However, it is not possible to relate the resulting integral to the approximate solution of ψ_L (unless the screen is very small). To do so we have to divide the screen into small rectangular segments, see Fig. 5, $D_{mn} = \{(y, z); |y - y_m| \leq y_D \text{ and } |z - z_n| \leq z_D\}$, so that

$$\psi_{k\ell}^{\text{Diff}} = \sum_{m=1}^M \sum_{n=1}^N \psi_{k\ell}^{mn} \quad (55a)$$

where

$$\psi_{k\ell}^{mn} = C \iint_{D_{mn}} \iint_{\Omega_2} \iint_{\Omega_1} A_{1k} A_{2\ell} F_0 \alpha_2 d\Omega_1 d\Omega_2 dy dz. \quad (55b)$$

In the subsequent analysis we confine the discussion to ψ_{22}^{mn} . Now interchange the order of integration in Eq. (55b) and integrate with respect to y and z :

$$\psi_{22}^{mn} = 4 y_D z_D^C \iint_{\Omega_2} \iint_{\Omega_1} A_{12} A_{22} E F_1 \alpha_2 d\Omega_1 d\Omega_2, \quad (56a)$$

where

$$E = E_1[\vec{r}_{mn} - \vec{r}_S^{(2)}] E_2[\vec{r}_{mn} - \vec{r}_R^{(2)}], \quad (56b)$$

$$F_1 = \frac{\sin[k_0 y_D(\beta_1 + \beta_2)]}{k_0 y_D(\beta_1 + \beta_2)} \times \frac{\sin[k_0 z_D(\gamma_1 + \gamma_2)]}{k_0 z_D(\gamma_1 + \gamma_2)} \quad (56c)$$

and

$$\vec{r}_{mn} = (0, y_m, z_n). \quad (56d)$$

When evaluating the reflected part of the field from an impedance boundary [cf. Eq. (41a)] it is advantageous to introduce a new polar axis (new variables of integration) through the image source and the receiver²⁷, as the essential contribution is in this direction. If we do so with both ψ_{12} and ψ_{22} , we may show that F_1 is dominated by "plane waves" in the direction source (or receiver) to segment and approximate F_1 by a suitable function. This is done in two steps as follows:

As a first step, introduce new variables of integration, ϕ'_1 , ϕ'_2 , γ'_1 and γ'_2 , according to the relations

$$\alpha'_1 = \alpha_1 \cos(\phi_0) + \beta_1 \sin(\phi_0), \quad (57a)$$

$$\beta'_1 = \beta_1 \cos(\phi_0) - \alpha_1 \sin(\phi_0), \quad (57b)$$

$$\gamma'_1 = \gamma_1, \quad (57c)$$

where

$$\cos(\phi_0) = -x_S/R_0, \quad (58a)$$

$$\sin(\phi_0) = (y_m - y_S)/R_0, \quad (58b)$$

$$R_0 = [x_S^2 + (y_m - y_S)^2]^{1/2} \quad (58c)$$

and where α'_1 and β'_1 are related to ϕ'_1 and γ'_1 by relations similar to Eqs. (52c) and (52d). The new variables ϕ'_2 and γ'_2 have been defined in a similar way.

As a second step, introduce new variables, ϕ''_1 , ϕ''_2 , γ''_1 and γ''_2 , defined by

$$\alpha''_1 = \alpha'_1 \cos(\theta'_0) - \gamma'_1 \sin(\theta'_0), \quad (59a)$$

$$\beta''_1 = \beta'_1, \quad (59b)$$

$$\gamma''_1 = \gamma'_1 \cos(\theta'_0) + \alpha'_1 \sin(\theta'_0), \quad (59c)$$

where

$$\cos(\theta'_0) = (z_n + z_S)/R_{12} \quad ; \quad 0 \leq \theta'_0 < \pi/2, \quad (60a)$$

$$R_{12} = |\vec{r}_{mn} - \vec{r}_S^{(2)}| \quad (60b)$$

and similar expressions for ϕ''_2 and γ''_2 . It is easily seen that the domain of integration for Ω'_1 (corresponds to γ'_1 and ϕ'_1) and Ω'_2 is the same as for Ω_1 and Ω_2 . It is far from trivial that this is true for Ω''_1 and Ω''_2 too, but it has originally been shown by Weyl²⁶. The additional trouble caused by the poles at $\gamma_1 = -v$ and at $\gamma_2 = -v$ is avoided by using the contour L_1 .²⁷

These two steps enable us to write the integral of Eq. (56a) as

$$\psi_{22}^{mn} = 4 y_D z_D^C \iint_{\Omega''_2} \iint_{\Omega''_1} A_{12} A_{22} E F_1 \alpha_2 d\Omega''_1 d\Omega''_2, \quad (61a)$$

where now E , cf. Eq. (56b), takes the simple form

$$E = \exp[ik_0(\gamma''_1 R_{12} + \gamma''_2 R_{22})] \quad (61b)$$

with

$$R_{22} = |\vec{r}_{mn} - \vec{r}_R^{(2)}|. \quad (61c)$$

Somehow we want to relate the integral of Eq. (61a) to the approximate solutions of $\Psi_L(\vec{r}_0|\vec{r}_S)$ and $\Psi_L(\vec{r}_R|\vec{r}_0)$. We confine the discussion to Ψ_{12} . First we introduce a new contour L_2'' for the variable γ_1'' . L_2'' is defined similarly to L_2 , but with γ_0' [obtained from Eq. (18a) by changing θ_0 to θ_0'] instead of γ_0 , see Fig. 4. (This is possible as the poles of A_{12} in the γ_1'' -plane have a real part smaller than the pole at $\phi_2'' = 0$, i.e. the pole at γ_0'). With an argument similar to the one in subsection A [leading to Eq. (46)] we conclude that in the interesting frequency range the essential contribution to the integral originates from $\gamma_1'' \cong 1$ and — after a similar discussion — from $\gamma_2'' \cong 1$. We therefore put $\gamma_1'' = \gamma_2'' = 1$ in F_1 and find that [e.g. note that $\beta_1 = (y_m - y_S)/R_{12}$ for $\gamma_1'' = 1$]

$$\Psi_{22}^{mn} \cong 8 y_D z_D^F \Psi_{12}(\vec{r}_{mn}|\vec{r}_S) \partial \Psi_{22}(\vec{r}_R|\vec{r}_{mn}) / \partial x_R, \quad (62a)$$

$$F = \frac{\sin(k_0 y_D A)}{k_0 y_D A} \times \frac{\sin(k_0 z_D B)}{k_0 z_D B} \quad (62b)$$

and

$$A = \frac{y_m - y_S}{R_{12}} + \frac{y_m - y_R}{R_{22}}, \quad (62c)$$

$$B = \frac{z_n + z_S}{R_{12}} + \frac{z_n + z_R}{R_{22}} \quad (62d)$$

The function F (equals F_1 at $\gamma_1'' = \gamma_2'' = 1$) takes into account the phase shifts over the segments D_{mn} for the "plane waves" of Ψ_L in the dominating direction source (or receiver) to D_{mn} and makes it possible to use much less segments than necessary with other methods of integration.

It is natural to seek an estimate of the relative error, ϵ_{22}^{mn} , of Ψ_{22}^{mn} caused by the assumption that $F_1(\gamma_1'', \gamma_2'') = F_1(1, 1)$. A suitable expansion of F_1 at $\gamma_1'' = \gamma_2'' = 1$ indicates that ϵ_{22}^{mn} is of the order of magnitude

$$\epsilon_{22}^{mn} \sim \max(D_{\text{Max}}, k_0 D_{\text{Max}}^2) / R_{\text{Min}}, \quad (63a)$$

where

$$D_{\text{Max}} = \max(y_D, z_D) \quad ; \quad R_{\text{Min}} = \min(R_{12}, R_{22}). \quad (63b)$$

The other terms, $\psi_{k\ell}^{mn}$, of the sum Eq. (50a) may be treated in a similar way. An additional trouble is that the direct fields ψ_{11} and ψ_{21} contain expressions of the type $E_1(x_S, y_S - y, z_S - z)$, which gives a number of separate cases when integrating with respect to z , due to the absolute value of $z_S - z$, see Eq. (52a). We may overcome this difficulty by noting that Eq. (51) still holds, if instead we consider $E_1(z_S - z, y_S - y, x_S)$, which is possible with the direct fields ψ_{11} and ψ_{21} , as the expressions for these fields do not contain reflection coefficients and, accordingly, no poles. Performing the same steps with all the $\psi_{k\ell}^{mn}$ -terms as with ψ_{22}^{mn} , very similar results are obtained both for the approximate expressions of $\psi_{k\ell}^{mn}$ [see Eqs. (65) and (67)] and for the relative errors, $\epsilon_{k\ell}^{mn}$, of these terms.

One problem remains: What is the number of segments needed, i.e., how large should M and N in Eq. (55a) be? When integrating numerically, M and N are often increased until a satisfactory accuracy is obtained. However, for our specific problem the relative error of $\psi_{\text{Hard}}^{\text{Diff}}$ may be estimated, and this estimate can be used to reduce the number of segments considerably. In fact we may even find out M and N before the integration procedure is started. To do so we must first see which quantities that are of interest. Compared to other errors the error in the approximate solution of ψ_L is small; ψ_L may be considered exact. With a screen present there are mainly two quantities — the level relative free field, $20 \times \log |\psi_{\text{Hard}}^{\text{P.O.}} / \psi_I|$, and the insertion loss, $20 \times \log |\psi_L / \psi_{\text{Hard}}^{\text{P.O.}}|$. The number of segments needed does not depend on the level relative free field, but on the insertion loss. For a fixed relative error, ϵ , of the diffracted field $\psi_{\text{Hard}}^{\text{Diff}}$, the relative error of the total field $\psi_{\text{Hard}}^{\text{P.O.}}$ increases when $|\psi_{\text{Hard}}^{\text{P.O.}}|$ decreases, i.e. when the insertion loss increases [see Eq. (64f) below]. We will now show that it is possible to find a relation between the insertion loss and M and N . We start by relating M and N to ϵ .

The relative error, ϵ , of $\psi_{\text{Hard}}^{\text{Diff}}$ is assumed to be of the same order of magnitude as the terms $\epsilon_{kl}^{\text{mn}}$, see Eq. (63), and it is convenient to put $R_{\text{Min}} = \min(|x_S|, |x_R|)$, as R_{Min} cannot be less than $\min(|x_S|, |x_R|)$. Further, as the size of the segments should now roughly be

$$y_D = z_D \leq \min[\epsilon R_{\text{Min}}, (\epsilon R_{\text{Min}}/k_0)^{1/2}], \quad (64a)$$

if the relative error of $\psi_{\text{Hard}}^{\text{Diff}}$ shall not exceed ϵ , we choose M and N as

$$M = 1 + \text{Int}[\frac{1}{2}(Y_1 - Y_0)/Y_{\text{Min}}] \quad (64b)$$

and

$$N = 1 + \text{Int}[\frac{1}{2} Z_1/Z_{\text{Min}}], \quad (64c)$$

where $\text{Int}[\]$ denotes the integer part and where

$$Y_{\text{Min}} = Z_{\text{Min}} = \min[\epsilon X_{\text{Min}}, (\epsilon X_{\text{Min}}/k_0)^{1/2}] \quad (64d)$$

and

$$X_{\text{Min}} = \min(|x_S|, |x_R|). \quad (64e)$$

This also defines y_D and z_D , see Eqs. (68a) and (68b) below.

Once ϵ is known, the entire solution gives a fixed result. How should ϵ be chosen? A suitable expansion of the relation between $\psi_{\text{Hard}}^{\text{Diff}}$ and the insertion loss [Eq. (29)] shows that the difference ΔIL between the insertion loss for $\epsilon = 0$ (M and N approach infinity) and for $\epsilon \neq 0$, when $\Delta\text{IL} \gtrsim 3$ dB, is

$$\Delta\text{IL} \sim \epsilon \cdot 8.7 \cdot 10^{\text{IL}_M/20} \quad (\text{dB}) \quad (64f)$$

If the calculated insertion loss $\text{IL}_{\text{Hard}}^{\text{P.O.}}$ is less than IL_M . Eq. (64f) presupposes that the insertion loss is large enough to make $\psi_{\text{Hard}}^{\text{Diff}} \approx$

$-\Psi_L$, which is the case (approximately) when $IL_{\text{Hard}}^{\text{P.O.}} \geq 10$ dB. A rough estimate for the case when $IL_{\text{Hard}}^{\text{P.O.}} < 10$ dB may be obtained from Eq. (64f) by putting $IL_M = 10$ dB.

This concludes the analysis. Once we have decided the accuracy, ΔIL , and the maximum insertion loss considered, IL_M , then ϵ is known through Eq. (64f) and M and N through Eqs. (64b)-(64e).

The result in Eq. (62a) can also be obtained by noting that the integrand of Eq. (50b) contains factors of the type $\exp[ik_0(|\vec{r}_0 - \vec{r}_S^{(k)}| + |\vec{r}_0 - \vec{r}_R^{(l)}|)]$. Expand these factors near $\vec{r}_0 = \vec{r}_{mn}$ and integrate over D_{mn} with respect to y and z . Then we arrive at Eq. (62a) directly. To be sure of the influence of the poles in A_{12} and A_{22} we have not chosen this way.

D. The integral — result

The analysis of the preceding subsection shows that it is possible to write the physical optics solution, Eqs. (22) and (23), as (the soft screen case is obtained by exchanging $[\Psi_{1k}^{mn} \partial \Psi_{2l}^{mn} / \partial x_R]$ in Eq. (65c) for $[-\Psi_{2l}^{mn} \partial \Psi_{1k}^{mn} / \partial x_S]$):

$$\Psi_{\text{Hard}}^{\text{P.O.}}(\vec{r}_R | \vec{r}_S) = \Psi_L + \Psi_{\text{Hard}}^{\text{Diff}}, \quad (65a)$$

where (Ψ_{kl}^{mn}) is redefined below)

$$\Psi_{\text{Hard}}^{\text{Diff}} = \sum_{k=1}^2 \sum_{l=1}^2 \Psi_{kl}^{\text{Diff}}, \quad (65b)$$

$$\Psi_{kl}^{\text{Diff}} = 8 y_{D^z D} \sum_{m=1}^M \sum_{n=1}^N F \Psi_{1k}^{mn} \partial \Psi_{2l}^{mn} / \partial x_R \quad (65c)$$

$$\Psi_{11}^{mn} = -(4\pi R_{11})^{-1} \exp(ik_0 R_{11}), \quad (66a)$$

$$\Psi_{21}^{mn} = -(4\pi R_{21})^{-1} \exp(ik_0 R_{21}), \quad (66b)$$

$$\psi_{12}^{mn} = \psi_L(0, y_m, z_n | x_S, y_S, z_S) - \psi_{11}^{mn}, \quad (66c)$$

$$\psi_{22}^{mn} = \psi_L(x_R, y_R, z_R | 0, y_m, z_n) - \psi_{21}^{mn}. \quad (66d)$$

$$F = \frac{\sin(k_0 y_D A)}{k_0 y_D A} \times \frac{\sin(k_0 z_D B)}{k_0 z_D B}, \quad (67a)$$

$$A = \frac{y_m - y_S}{R_{1k}} + \frac{y_m - y_R}{R_{2l}}, \quad (67b)$$

$$B = \frac{z_n - K_k z_S}{R_{1k}} + \frac{z_n - K_l z_R}{R_{2l}}, \quad (67c)$$

$$R_{1k}^2 = x_S^2 + (y_m - y_S)^2 + (z_n - K_k z_S)^2, \quad (67d)$$

$$R_{2l}^2 = x_R^2 + (y_m - y_R)^2 + (z_n - K_l z_R)^2, \quad (67e)$$

$$K_l = \begin{cases} 1 & ; \quad l = 1, \\ -1 & ; \quad l = 2, \end{cases} \quad (67f)$$

and

$$y_D = (Y_1 - Y_0)/(2M), \quad (68a)$$

$$z_D = Z_1/(2N), \quad (68b)$$

$$y_m = (2m - 1)y_D + Y_0, \quad (68c)$$

$$z_n = (2n - 1)z_D. \quad (68d)$$

M and N are related to the relative error ϵ of $\psi_{\text{Hard}}^{\text{Diff}}$ as

$$M = 1 + \text{Int}\left[\frac{1}{2}(Y_1 - Y_0)/Y_{\text{Min}}\right] \quad (69a)$$

and

$$N = 1 + \text{Int}\left[\frac{1}{2} Z_1/Z_{\text{Min}}\right], \quad (69b)$$

where $\text{Int}[\]$ denotes the integer part,

$$Y_{\text{Min}} = Z_{\text{Min}} = \min[\epsilon X_{\text{Min}}, (\epsilon X_{\text{Min}}/k_0)^{1/2}] \quad (69c)$$

and

$$X_{\text{Min}} = \min(|x_S|, |x_R|). \quad (69d)$$

ϵ may be estimated before the integration of $\psi_{\text{Hard}}^{\text{Diff}}$ is made. To do so we must decide the largest acceptable error for the insertion loss, ΔIL , and the largest insertion loss to be considered, IL_M . If $\text{IL}_{\text{Hard}}^{\text{P.O.}} < \text{IL}_M$, the error of $\text{IL}_{\text{Hard}}^{\text{P.O.}}$ is less than ΔIL , if $\text{IL}_{\text{Hard}}^{\text{P.O.}} \geq \text{IL}_M$, the error is larger (for such cases one either takes IL_M as a conservative estimate of $\text{IL}_{\text{Hard}}^{\text{P.O.}}$ or increases IL_M and recalculates $\text{IL}_{\text{Hard}}^{\text{P.O.}}$). We choose ϵ as ($\Delta\text{IL} \approx 3$ dB)

$$\epsilon = \begin{cases} 0.11 \Delta\text{IL} 10^{-\text{IL}_M/20} & ; \text{IL}_M \geq 10 \\ 0.11 \Delta\text{IL} 10^{-0.5} & ; \text{IL}_M < 10, \end{cases} \quad (69e)$$

$$\epsilon = \begin{cases} 0.11 \Delta\text{IL} 10^{-0.5} & ; \text{IL}_M < 10, \end{cases} \quad (69f)$$

where IL_M and ΔIL are given in dB.

$\psi_{\text{Hard}}^{\text{P.O.}}$ may thus be evaluated by first deciding ΔIL and IL_M . This gives us ϵ via Eqs. (69e) and (69f), which in its turn gives M and N via Eqs. (69a)-(69d) and finally $\psi_{\text{Hard}}^{\text{P.O.}}$ by Eqs. (65a)-(68d).

V. PHYSICAL OPTICS SOLUTION — RESULTS AND COMMENTS

We are now ready to give the entire solution in a numerically suitable form. We confine ourselves to the case with a hard screen — the soft screen gives similar expressions. Two quantities are of interest: The level relative free field,

$$L_{\text{Hard}}^{\text{P.O.}} = 20 \times \log |\Psi_{\text{Hard}}^{\text{P.O.}} / \Psi_{\text{I}}| \quad (70)$$

and the corresponding insertion loss,

$$IL_{\text{Hard}}^{\text{P.O.}} = 20 \times \log |\Psi_{\text{L}} / \Psi_{\text{Hard}}^{\text{P.O.}}|. \quad (71)$$

$\Psi_{\text{Hard}}^{\text{P.O.}}$ is evaluated with the relations of Sec. IV:D [Eqs. (65)-(69)]. In these equations there are two functions that have been treated previously; $\partial \Psi_{\text{L}} / \partial x_{\text{R}}$, which is evaluated with the results of Sec. IV:B, [Eqs. (47a)-(47d)], and Ψ_{L} , which is evaluated with the results of Sec. II:B, [Eqs. (10)-(18)]. All geometrical quantities are found in Fig. 2 and the ground properties — defined through the constants a , b , c and d , see Eq. (10) — are estimated by making experiments without the screen as suggested in Sec. II.

It is suggested that both $\Psi_{\text{Hard}}^{\text{P.O.}}$ and $\Psi_{\text{Soft}}^{\text{P.O.}}$ are calculated. This is easily done, because when Ψ_{1k}^{mn} and Ψ_{2L}^{mn} have been calculated, $\partial \Psi_{1k}^{mn} / \partial x_{\text{S}}$ and $\partial \Psi_{2L}^{mn} / \partial x_{\text{R}}$ may be calculated with negligible effort. Reciprocity is violated to the same amount as $\Psi_{\text{Hard}}^{\text{P.O.}}$ and $\Psi_{\text{Soft}}^{\text{P.O.}}$ differ. Comparisons give a possibility to see when the physical optics solution fails.

Limitations of the solution are (a) both the source and the receiver must be a few wavelengths from the screen, so that the approximate solution of Ψ_{L} , Eq. (19), is valid, (b) the source and the receiver must be "far" away from the aperture ($x = 0$), so that the approximate solution of $\partial \Psi_{\text{L}} / \partial x_{\text{R}}$, Eq. (47), is valid and, finally, (c) experiments have shown — reciprocity is violated — that we cannot use the solution for grazing incidence [the angle in the x,y -plane between the line source-receiver and the normal to the screen should not exceed, say, 60 deg., see also (b)]. These restrictions are no practical restrict-

ions; in most cases the solution is very accurate, and the experiments of Secs. VI and VII will confirm this.

The only problem left is to decide the largest error acceptable, ΔIL , and the largest insertion loss considered, IL_M . If ΔIL is chosen to 2 dB, then ϵ is 0.07 and 0.02 if IL_M is 10 and 20 dB respectively, see Eqs. (69e) and (69f). In many practical cases we know by experience that the insertion loss does not exceed, say, 10 dB. This indicates that $\epsilon = 0.05$ is sufficient ($\Delta IL \approx 1.5$ dB). Then the segments of the screen (used when integrating) are squares with an approximate area of 2.2 m^2 (100 Hz) and 0.2 m^2 (1000 Hz), if neither the source nor the receiver are closer to the screen than 20 m. Note that if $L_{\text{Hard}}^{\text{P.O.}}$, Eq. (70), is the desired quantity, we should still choose ϵ with respect to the insertion loss, as there are no relations between $L_{\text{Hard}}^{\text{P.O.}}$ and M or N .

To show the numerical gain obtained by using the method in Sec. IV:D, we compare this type of integration with Simpson's rule [with Eq. (68c) exchanged for $y_m = 2m y_D + Y_0$, $m = 0, \dots, M$ and similarly for Eq. (68d)] and with trapezoidal integration [$F = 1$ in Eq. (67a)]. This is done in Table I, and the results make it evident that it is possible to use considerably less segments (at least a factor 10 less) when the method in Sec. IV:D is used. The reason is that normal methods for numerical integration give rise to unusable results if the number of integration points is small (for $\epsilon = 0.1$, N varies between 2 and 5 in Table I). The solution in Sec. IV:D brings the execution effort down to a reasonable level. The entire insertion loss response, 100-1000 Hz, with the method from Sec. IV:D in Table I was done in 8 ($\epsilon = 0.1$) and 75 ($\epsilon = 0.01$) seconds respectively (with a UNIVAC 1108 computer). As 0.05 will do in most cases, and as the calculation effort is proportional to the area of the screen, the solution should be quite useful for the evaluation of the insertion loss or the levels with screen. In connection with Table I, note that the approximate ΔIL in Eq. (64f) agrees well with the real difference, both for $\epsilon = 0.1$ and $\epsilon = 0.01$. In Table I it can also be seen that ΔIL is proportional to ϵ (compare $\epsilon = 0.1$ and $\epsilon = 0.01$) as suggested by Eq. (64f).

Freq. (Hz)	ΔIL_S (dB)		ΔIL_T (dB)		ΔIL (dB)		IL (dB)
100	-1.4	0.0	-1.8	-0.1	-0.8	0.0	7.0
126	-13.1	0.0	5.8	0.1	-2.2	-0.1	14.0
159	-11.4	0.0	-4.3	-0.1	-2.2	-0.2	13.5
200	-8.2	0.0	-9.1	0.2	-1.9	0.2	13.2
252	-12.1	0.1	-8.1	0.0	1.3	0.2	11.5
317	-8.4	0.2	-7.0	0.1	-0.5	-0.1	7.7
400	-9.4	0.3	-11.0	-0.2	1.6	-0.1	5.3
504	-16.8	0.4	-16.4	0.0	0.9	-0.1	6.6
635	-18.1	-4.6	-15.8	-0.8	-1.7	0.1	7.4
800	-23.3	-12.7	-16.2	1.0	0.4	0.0	11.6
1008	-23.3	-10.0	-15.2	-0.2	-0.6	0.1	10.3
ϵ	0.1	0.01	0.1	0.01	0.1	0.01	0.003

TABLE I. Comparisons between different methods of integration. IL is the insertion loss calculated according to Eqs. (65)-(69) with $\epsilon = 0.003$. Eq. (64f) indicates that IL is correct within ± 0.1 dB. ΔIL_S is the difference between the insertion loss calculated with Simpson's rule and IL. ΔIL_T is the difference between the insertion loss calculated with trapezoidal integration [Eqs. (65)-(69) with $F = 1$] and IL. ΔIL is the difference between the insertion loss calculated with Eqs. (65)-(69) and IL. The screen is acoustically hard. The soft screen gives similar results.

Ground: $a = 0.25$, $b = 10^0$, $c = 7500$ Hz and $d = 425$ Hz, see Eq. (10). Geometry: $(x_S, y_S, z_S) = (20, 0, 0.1)$ m; $(x_R, y_R, z_R) = (-20, 0, 2)$ m; $(Y_0, Y_1, Z_1) = (-12.5, 12.5, 3)$ m, see Fig. 2.

VI. MODEL EXPERIMENTS

A. General

In order to check and illuminate some aspects of the theory, a model experiment was performed. This is advantageous for several reasons, mainly because it is easy to get hold of the point impedance, eliminate meteorological conditions and study different types of screens.

To start with, the ground was simulated by a 3 mm thick felt on a $3.3 \times 3.2 \text{ m}^2$ hard backing. This has been examined previously³⁰. It was found that the felt could be described by a point impedance in the sense that in the frequency range 0.5-50 kHz the measured frequency responses agreed very well with the theoretical responses for a single choice of the parameters of v : a , b , c and d [see Eq. (10)]. Thus the experiment confirmed the relation, previously mentioned in Sec. II:B, between the impedance boundary and the porous layer. The present study is a continuation of sound propagation above a plane ground. The sound field close to the felt exhibits some interesting features, viz. in the frequency range where the field is dominated by surface waves, and in the frequency range where the ground attenuation is very strong [see e.g. Fig. 6(a) at 2 and 20 kHz respectively]. Further details of the experimental set-up are given in the model study cited above³⁰.

The screens were made of aluminium plates (1 mm thick). In a preliminary investigation it was found that in the range 0.5-5 kHz vibrations in the plate, or possibly other phenomena, affected the diffraction. To avoid these problems the aluminium plate was replaced by a 14 mm thick screen (12 mm wood, on each side covered with 1 mm aluminium plates) in the range 0.5-5 kHz.

The following experiments are made to compare the physical optics solution in Sec. V with experimental data. Particularly we shall study the influence of the boundary conditions of the ground, the receiver height, the height, length and side edge position of the screen, and the angle of incidence of the sound field. We shall also study some different types of barriers, to indicate the influence of its shape. In a separate report³¹ the full experimental material is presented.

Throughout this work the data has been presented in the form of

frequency sweeps. The reason is that in most cases the insertion loss is strongly dependent on the frequency, which makes information with this variable fixed difficult to interpret.

B. Dependence on ground impedance

As mentioned in the introduction the very strong need to develop a solution is inherent in the fact that at certain frequencies the ground attenuation will reduce the effect of a barrier to such a large extent that calculations based on free field screens with a perfectly hard boundary do not necessarily have anything to do with reality. This is illustrated in Fig. 6(b), which shows the insertion loss responses for both the perfectly hard ground and the porous layer. Fig. 6(a) shows the frequency responses with and without the screen, and as these responses were taken for the geometry for which the admittance was estimated, the agreement is as good as possible. When comparing these responses with the insertion loss responses, one immediately sees that the insertion loss for the hard boundary is larger than for the impedance boundary, approximately in the frequency range where the levels without screen for the hard boundary are larger than for the impedance boundary. It is difficult to show, whether this is a general tendency, but it is interesting to note that the theory is capable of showing this feature too. One should also note that in the range where the levels are larger than 6 dB (at least at 2 and 2.5 kHz) it was found³² that the surface wave term [ψ_3 , see Eq. (14b)] dominates the field. Here it is tempting to interpret the extra insertion loss for the impedance boundary (compared to the hard boundary) as an obstruction for the surface waves. This is difficult to prove, but the field behind the screen does not exhibit any of the properties of the surface wave term — vertical or horizontal exponential decrease. In any case the agreement between experimental and theoretical insertion loss responses in Fig. 6 is satisfactory.

One more example of the dependence on ground impedance will be given in connection with the outdoor experiments, see Sec. VII.

C. Dependence on receiver height

We know that the insertion loss of a screen is small in the illuminated region — in most cases negligible — whereas the insertion loss in the shadowed region is difficult to predict. This is a consequence of the large variation of the dip in the frequency response for the impedance

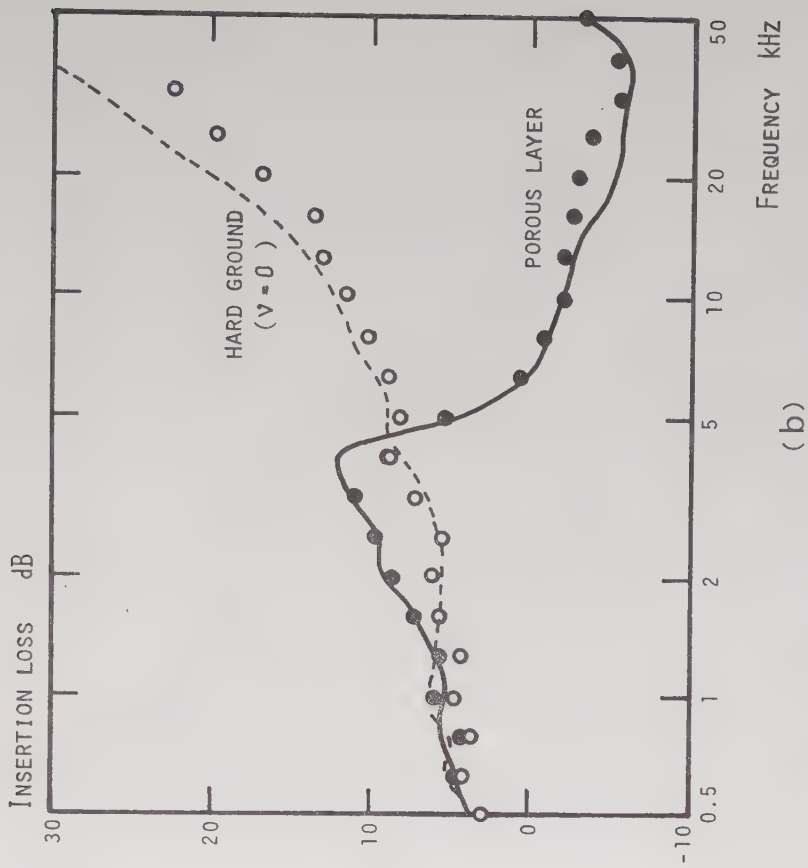
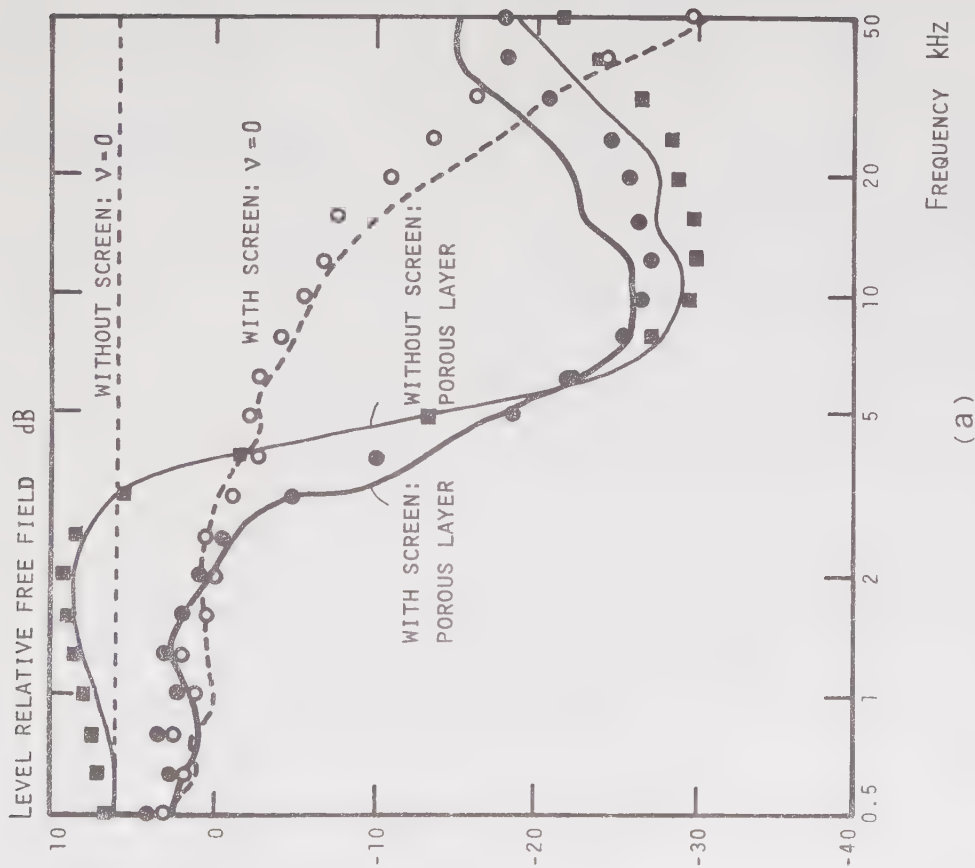


FIGURE 6. Frequency responses from a perfectly hard ground and a locally reacting porous layer with and without screen. Calculations according to Sec. V. Measured reference level: Hard ground without screen, which equals 6 dB rel. free field. Hard ground: $v = 0$ and impedance boundary: $a = 0.51$, $b = 0^\circ$, $c = 1.9 \times 10^5$ Hz and $d = 8700$ Hz. Solid and dashed lines, measured; open- and filled-circle line and squares, calculated. Geometry: $(x_S, y_S, z_S) = (1, 0, 0.01)$ m; $(x_R, y_R, z_R) = (-1, 0, 0.01)$ m; $(Y_0, Y_1, Z_1) = (-1, 1, 0.15)$ m, see Fig. 2. These somewhat extreme results are due to the low source and receiver heights.

boundary. An example is given in Fig. 7. We note that the screen is

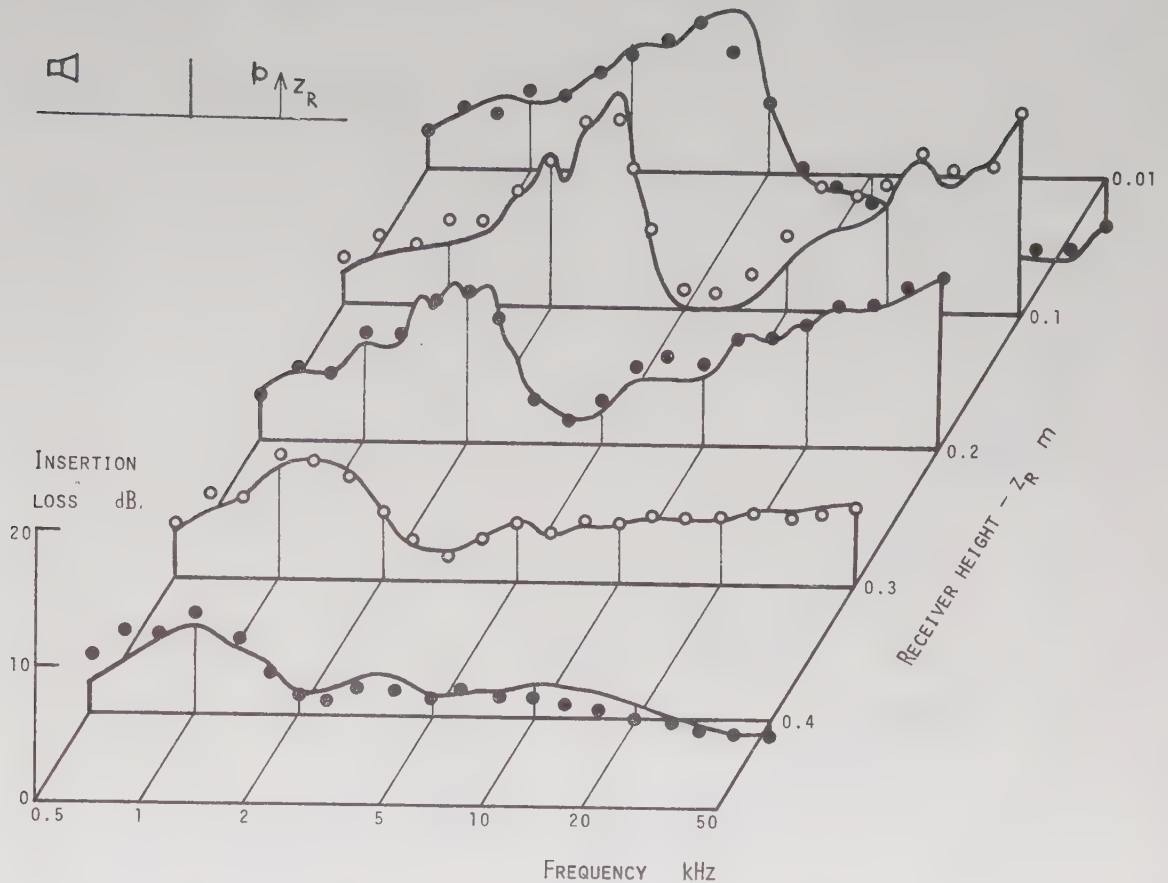


FIGURE 7. Insertion loss responses as a function of receiver height for a screen on a locally reacting porous layer, a felt. Calculations according to Sec. V. Solid line, measured; open- and filled-circle line, calculated.

Ground: $a = 0.51$, $b = 0^0$, $c = 1.9 \times 10^5$ Hz and $d = 8700$ Hz, see Eq. (10). Geometry: $(x_S, y_S, z_S) = (1, 0, 0.01)$ m; $(x_R, y_R, z_R) = (-1, 0, z_R)$ m; $(Y_0, Y_1, Z_1) = (-1, 1, 0.15)$ m, see Fig. 2.

effective in the entire shadowed region ($z_R < 0.29$ m) only at the low frequencies 1-4 kHz; below these frequencies the insertion loss rapidly vanishes. If the receiver is located not too close to the boundary, this is also true at the high frequencies — above 8 kHz. In a mid-region, 4-8 kHz, the ground attenuation at the receiver heights in the shadowed region is large and, as a consequence, the insertion loss is poor. If the source spectrum is dominated by frequencies in this range, the screen must be high to be effective.

Note that the insertion loss is approximately 6 dB at the line of sight source-upper edge-receiver ($z_R = 0.29$ m), as for a semi-infinite screen.

D. Dependence on screen height

We cannot expect any simple relation to account for the screen height. The ground properties are a function of the frequency and will affect the insertion loss. This is illustrated by Fig. 8. For most frequencies

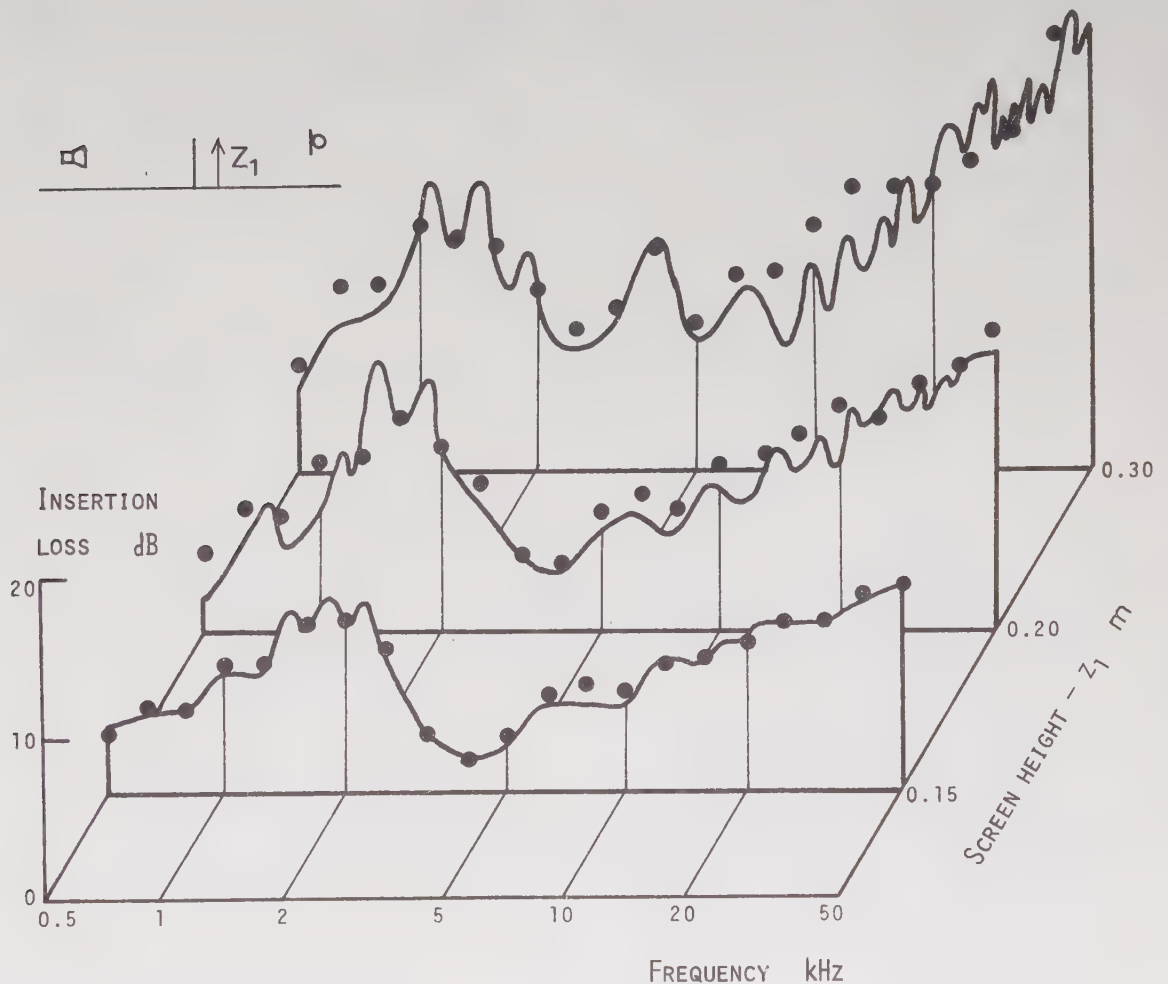


FIGURE 8. Insertion loss responses as a function of screen height for a screen on a locally reacting porous layer, a felt. Calculations according to Sec. V. Solid line, measured; filled-circle line, calculated.

Ground: $a = 0.51$, $b = 0^\circ$, $c = 1.9 \times 10^5$ Hz and $d = 8700$ Hz, see Eq. (10). Geometry: $(x_S, y_S, z_S) = (1, 0, 0.01)$ m; $(x_R, y_R, z_R) = (-1, 0, 0.2)$ m; $(Y_0, Y_1, Z_1) = (-1, 1, Z_1)$ m, see Fig. 2.

an increase of the screen height will increase the insertion loss. As a result of diffraction around the side edges of the screen, which is more important the higher the screen is, the insertion loss fluctuates rapidly with frequency in some frequency ranges. This ripple vanishes if the screen is sufficiently long, as shown in Fig. 9.

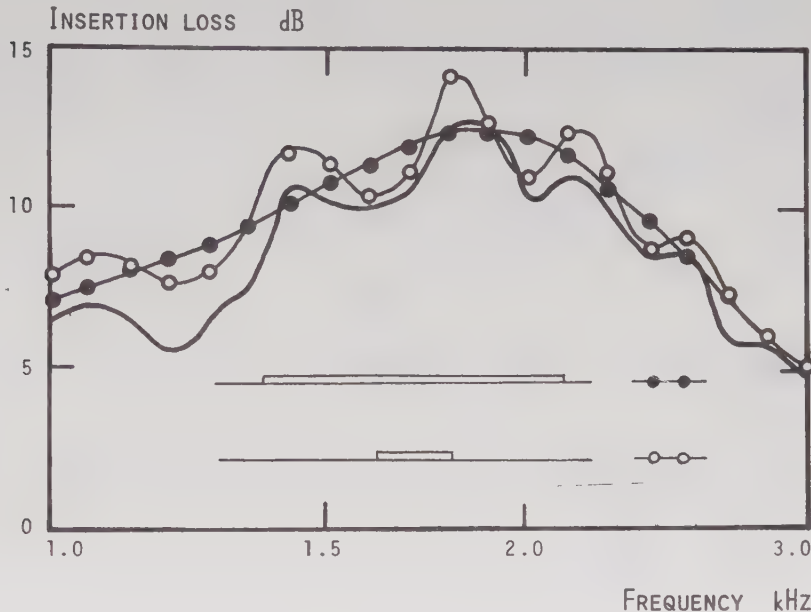


FIGURE 9. Diffraction around side edges. Detail from Fig. 8 for $Z_1 = 0.15$ m. Calculations according to Sec. V. Solid line, measured: $s = 1$ m; open-circle line, calculated: $s = 1$ m; filled-circle line, calculated: $s = 4$ m.

Ground: $a = 0.51$, $b = 0^\circ$, $c = 1.9 \times 10^5$ Hz and $d = 8700$ Hz, see Eq. (10). Geometry: $(x_S, y_S, z_S) = (1, 0, 0.01)$ m; $(x_R, y_R, z_R) = (-1, 0, 0.2)$ m; $(Y_0, Y_1, Z_1) = (-s, s, 0.15)$ m, see Fig. 2.

E. Dependence on screen length

This is an interesting aspect of the physical optics solution of Sec. V, as the other solutions previously mentioned cannot account for it (see Sec. III:D). An example is given in Fig. 10. One sees that at the high frequencies the average of the ripple for the short screen is essentially the same as for the long screen, which makes the difference small, at least for broad band noise. At low frequencies the diffraction around the two side edges may lead to important deviations from the results for the long screen.

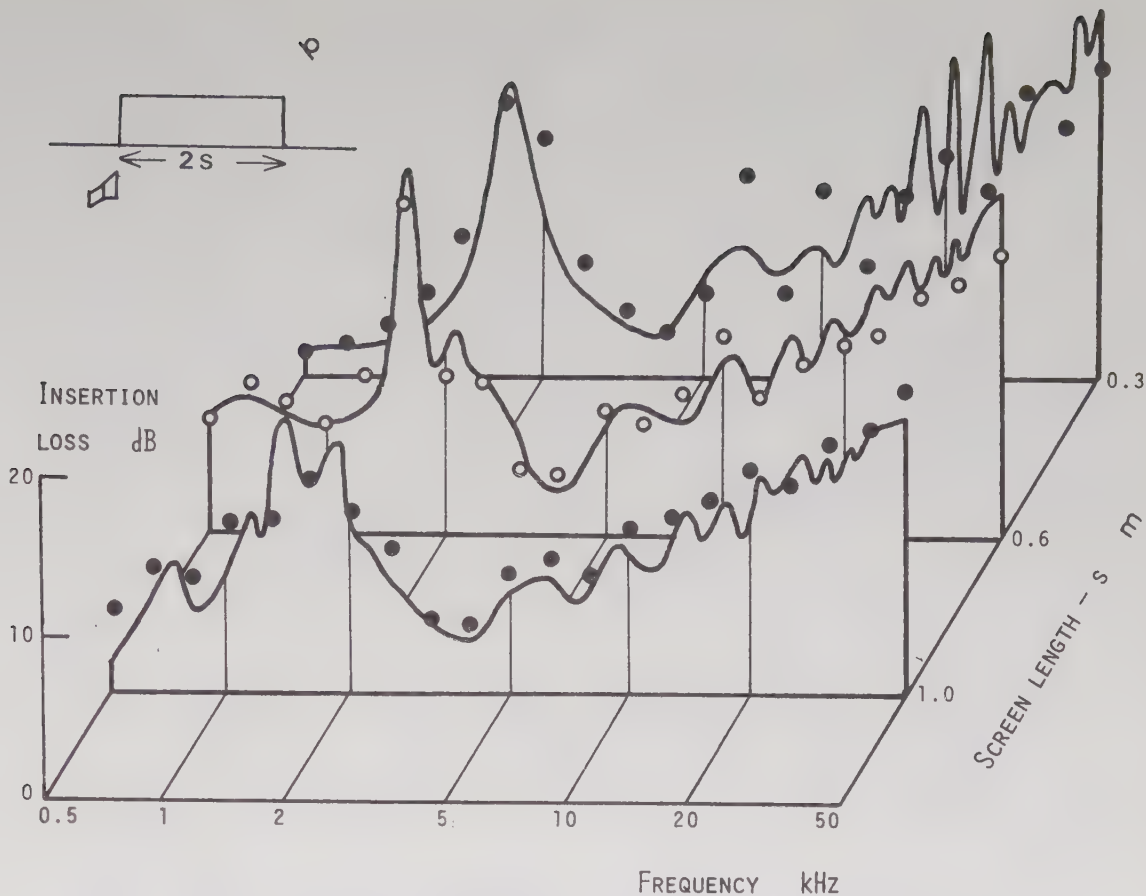


FIGURE 10. Insertion loss responses as a function of screen length for a screen on a locally reacting porous layer, a felt. Calculations according to Sec. V. Solid line, measured; open- and filled-circle line, calculated.

Ground: $a = 0.51$, $b = 0^\circ$, $c = 1.9 \times 10^5$ Hz and $d = 8700$ Hz, see Eq. (10). Geometry: $(x_S, y_S, z_S) = (1, 0, 0.01)$ m; $(x_R, y_R, z_R) = (-1, 0, 0.2)$ m; $(Y_0, Y_1, Z_1) = (-s, s, 0.2)$ m, see Fig. 2.

F. Dependence on side edge position

If, for a fixed position of the source and the receiver, we move the screen along the impedance boundary in the $x=0$ -plane, [see Fig. 2] we will expect the insertion loss to decrease. Fig. 11 shows an example.

The general conclusions seem to be that in the shadowed zone the diffraction around the side edge (at $s = -Y_0$, see Fig. 2) is small, if we are some screen heights within the shadowed region, and that the insertion loss rapidly goes to zero in the illuminated zone. At the line

of sight source-side edge-receiver ($s = 0$) the insertion loss is less than 6 dB in contrast to the line of sight source-upper edge-receiver in Fig. 7, as the screen is not infinitely high — we have diffraction around the side edge as well as at the upper edge in Fig. 11.

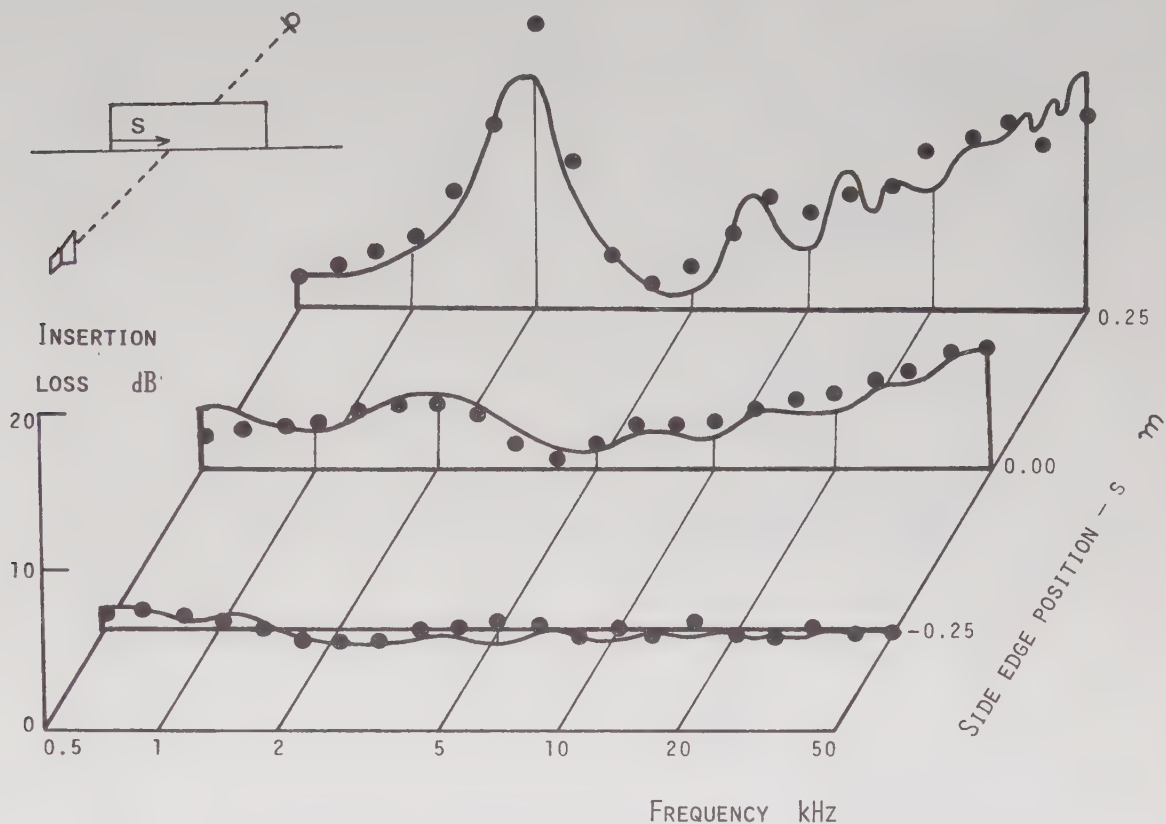


FIGURE 11. Insertion loss responses as a function of side edge position for a screen on a locally reacting porous layer, a felt. Calculations according to Sec. V. Solid line, measured; filled-circle line, calculated.

Ground: $a = 0.51$, $b = 0^0$, $c = 1.9 \times 10^5$ Hz and $d = 8700$ Hz, see Eq. (10). Geometry: $(x_S, y_S, z_S) = (1, 0, 0.01)$ m; $(x_R, y_R, z_R) = (-1, 0, 0.2)$ m; $(Y_0, Y_1, Z_1) = (-s, 2 - s, 0.15)$ m, see Fig. 2.

G. Dependence on angle of incidence

In many cases of practical interest the screen is not perpendicular to the line source-receiver. One does not expect any critical dependence for small deviations from this case. If the angle of incidence ϕ in the x, y -plane does not affect the insertion loss seriously, one may reduce

the number of parameters in prediction schemes. Fortunately, this seems to be the case, as one may note in Fig. 12. For angles in the range 0-60 degrees the insertion loss varies very little, apart from the weak

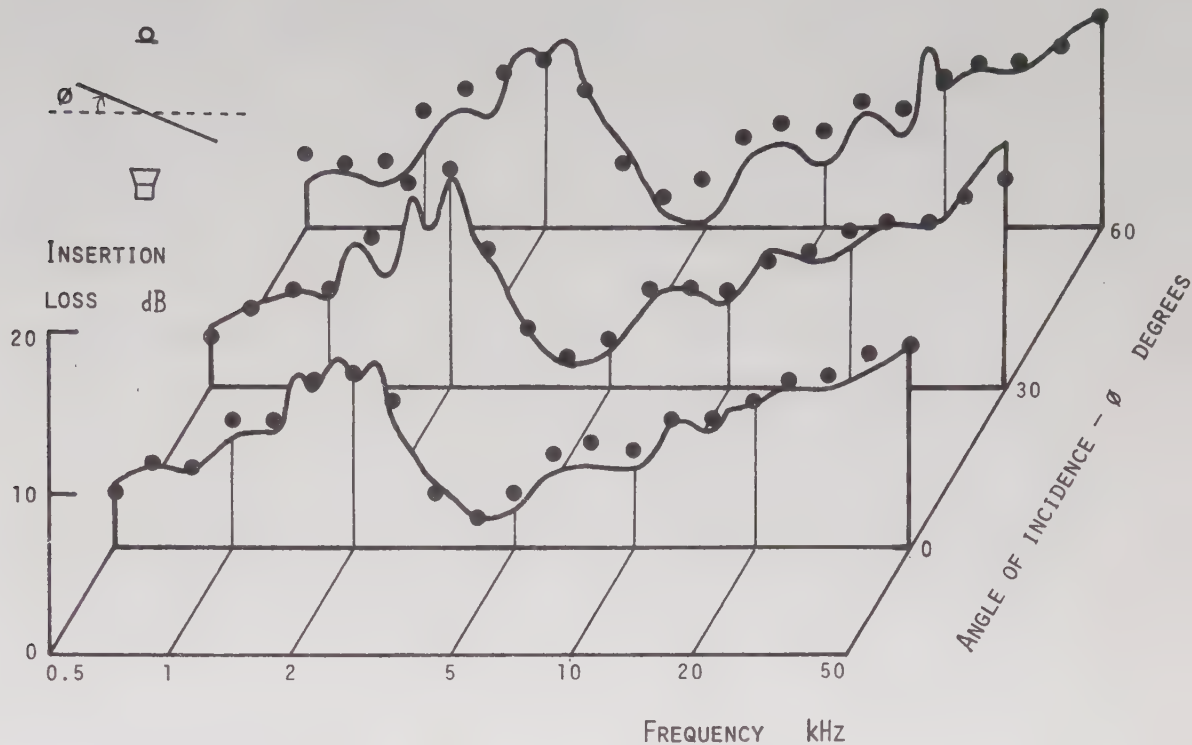


FIGURE 12. Insertion loss responses as a function of angle of incidence for a screen on a locally reacting porous layer, a felt. Calculations according to Sec. V. Solid line, measured; filled-circle line, calculated.

Ground: $a = 0.51$, $b = 0^\circ$, $c = 1.9 \times 10^5$ Hz and $d = 8700$ Hz, see Eq. (10). Geometry: $(x_S, y_S, z_S) = (\cos(\phi), \sin(\phi), 0.01)$ m; $(x_R, y_R, z_R) = (-\cos(\phi), -\sin(\phi), 0.2)$ m; $(Y_0, Y_1, Z_1) = (-1, 1, 0.15)$ m, see Fig. 2.

ripple in the frequency range 1-2 kHz, due to diffraction around the side edges. This conclusion seems to disagree with the conclusions made by Jonasson³³. However, it appears that he did not only vary the angle of incidence, but also the distance source-receiver. The weak dependence on ϕ can also be seen in the approximations leading to Lindblad's and Jonasson's solutions, see Sec. III:E. If the screen is turned an angle ϕ (not too large), Eq. (35) should instead read

$$\partial \psi_L / \partial x_R \cong -\cos(\phi) i k_0 \psi_L \quad (72)$$

and Eq. (36)

$$\psi_L \cong \psi_S \exp\{i k_0 \frac{1}{2} [(y \cos \phi)^2 + z^2] / x_S\}. \quad (73)$$

By using suitable new variables it is seen that this does not affect the right-hand side of Eq. (38), i.e. we find no ϕ -dependence of the insertion loss.

For angles larger than 60 degrees the solution cannot be used, as the insertion loss for the perfectly soft and the perfectly hard screens differ considerably and therefore the reciprocity condition is violated, see Eqs. (27) and (28). At 60 degrees this difference is at most 2 dB (125 Hz), but for other frequencies much less. As to the problem of reciprocity, large angles of incidence seem to be the only case where the physical optics solution fails.

H. Other types of barriers

For practical and aesthetical reasons it may be of interest to design the barrier in some other way than as a screen. In Fig. 13 some other types of barriers are studied. For the limited number of cases in Fig. 13 it seems as if deviations from the screen behaviour occur only in the low frequency region for the wedge-type barriers. For the thick barrier (thickness equals height) the deviation is small compared to the thin one, although it is several wavelengths thick, at least at the high frequencies. This seems to agree with extensive model tests for barriers on hard ground made by Ringheim³⁴. Apart from positions near the screen, his results indicate that the shape of the barrier has a marginal effect.

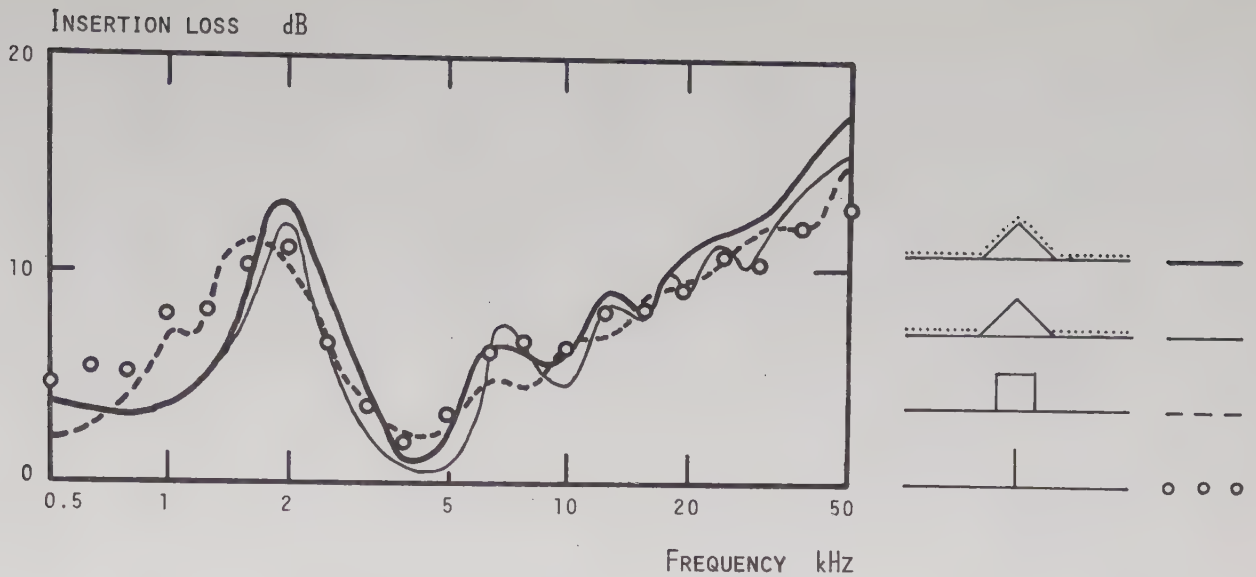


FIGURE 13. Some different types of barriers, equally high, on a locally reacting porous layer, a felt. Two right angle wedges: one with the same impedance boundary as the ground (solid line) and one with a perfectly hard boundary (solid thin line). One wide barrier; perfectly hard boundaries with height equal to breadth (dashed line). Compare with the screen in Fig. 12. Calculations for a screen (open-circle line) according to Sec. V.

Ground: $a = 0.51$, $b = 0^\circ$, $c = 1.9 \times 10^5$ Hz and $d = 8700$ Hz, see Eq. (10). Geometry: $(x_S, y_S, z_S) = (1, 0, 0.01)$ m; $(x_R, y_R, z_R) = (-1, 0, 0.2)$ m; $(Y_0, Y_1, Z_1) = (-1, 1, 0.15)$ m, see Fig. 2.

VII. OUTDOOR EXPERIMENTS

The outdoor experiments are not extensive compared to the model experiments. In order to see the influence of other phenomena than reflection and diffraction, and to check the theory for longer distances than is possible in the anechoic chamber, a short study was made. This experiment is a continuation of the one in Ref. 8, Sec. IX, where sound propagation above some plane outdoor surfaces without screens were studied. The reader is referred to the former study for further information about the instrumentation; the surfaces are the same. In the present study only screens on grassland are studied. The measurements with a screen were made three days later under the same meteorological conditions: no wind but temperature inversion. The positions of the source and the receiver were not exactly the same as without a screen, but this should not give better agreement than otherwise. The impedance was checked (with the method in Ref. 8, Sec. VII) but the limited data did not indicate that the time spread of measured frequency responses was larger than the spacial spread³⁵.

The screen was built up of elements $(1.5 \times 1 \text{ m}^2$ with thickness 0.015 m) of laminated wood, normally used to mould concrete. This choice allows us both to regard the screen as thin and to neglect transmission through it. The screen was 1.5 m high and its length was 25 m.

The first example is given in Fig. 14, taken from the geometry in which the impedance was estimated and with the screen inserted symmetrically between the source and the receiver. Fig. 14(a) gives an idea of the frequency response in the case with screen in comparison with the case without. As to the insertion loss, Fig. 14(b), the agreement is slightly less than in the model study, but still satisfactory.

Some examples, illustrating the insertion loss at longer distances, are given in Fig. 15. We see that although the agreement is less than at short distances [Fig. 14(b)] it is still satisfactory, especially when considering the temperature inversion. It should be noted that for the cases studied outdoors $[(x_S, x_R) = (10, -10), (20, -20) \text{ and } (20, -40) \text{ m}]$ with $z_R = 0.1, 0.5, 1 \text{ and } 2 \text{ m}$, see Ref. 31] it seems as if the insertion loss is slightly easier to predict than both the levels with and without screen. The difference between theory and measurements for the insertion

loss in the cases mentioned does not exceed 4.5 dB for any frequency in the range 0.1-1 kHz, whereas this difference for the levels relative free field may be up to 7 dB without screen and 9 dB with screen.

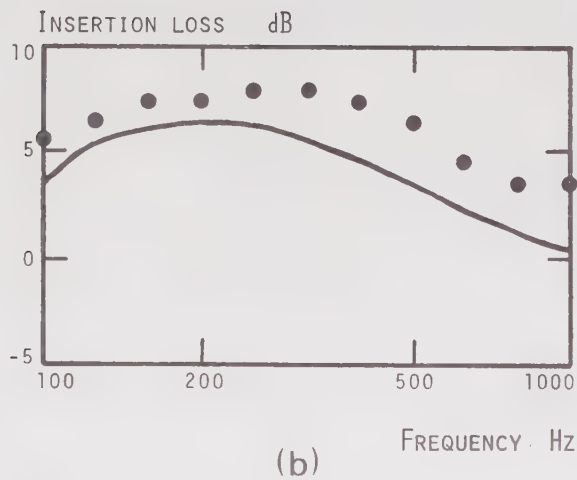
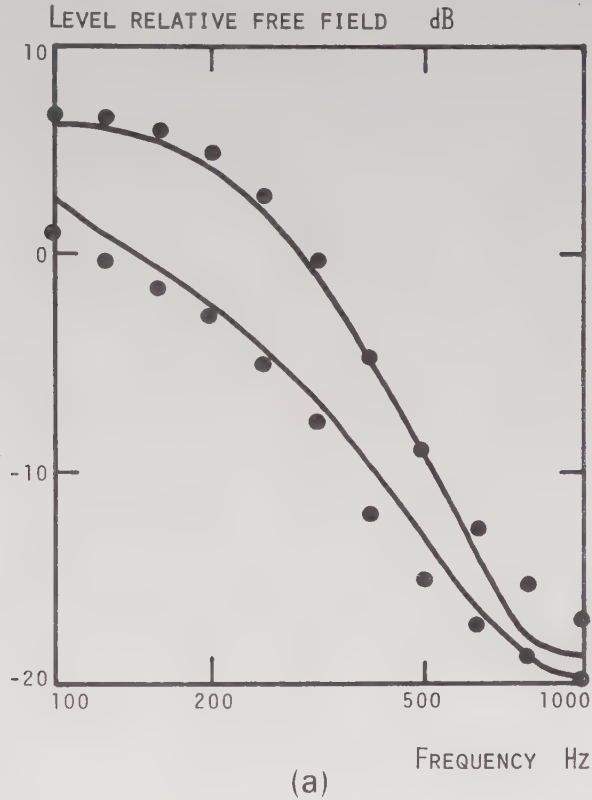


FIGURE 14. Frequency responses from an impedance boundary, grassland. Compare with the hard boundary and the felt, Fig. 6. Calculations according to Sec. V. Solid line, measured; filled-circle line, calculated.

Ground: $a = 0.25$, $b = 10^0$, $c = 7500$ Hz and $d = 425$ Hz, see Eq. (10). Geometry: $(x_S, y_S, z_S) = (10, 0, 0.1)$ m; $(x_R, y_R, z_R) = (-10, 0, 0.1)$ m; $(Y_0, Y_1, Z_1) = (-12.5, 12.5, 1.5)$ m, see Fig. 2.

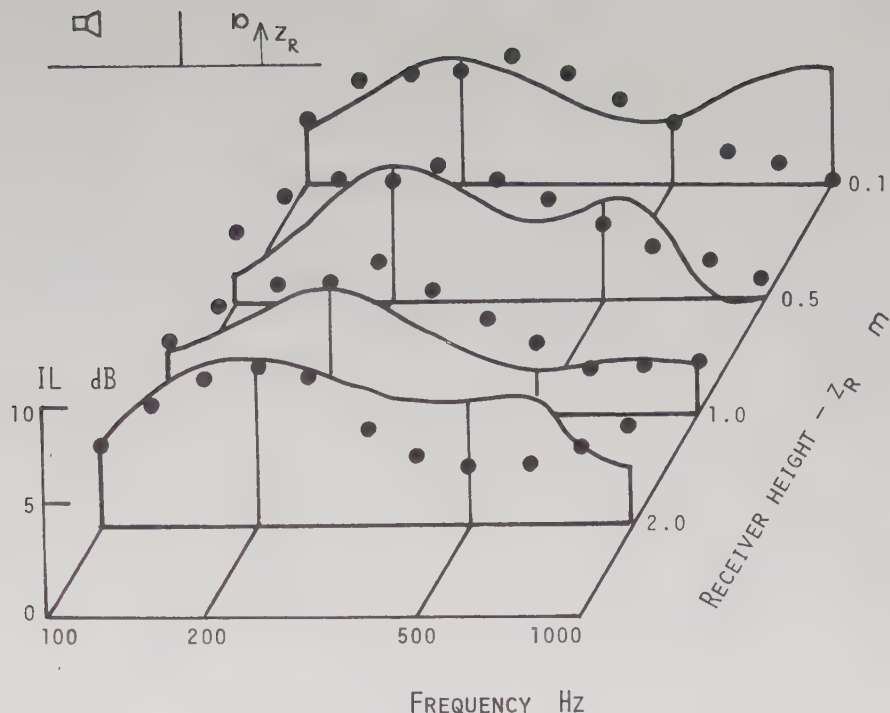


FIGURE 15. Insertion loss responses as a function of receiver height for a screen above an impedance boundary, grassland. Calculations according to Sec. V. Solid line, measured; filled-circle line, calculated.

Ground: $a = 0.25$, $b = 10^0$, $c = 7500$ Hz and $d = 425$ Hz, see Eq. (10). Geometry: $(x_S, y_S, z_S) = (20, 0, 0.1)$ m; $(x_R, y_R, z_R) = (-20, 0, z_R)$ m; $(Y_0, Y_1, Z_1) = (-12.5, 12.5, 1.5)$ m, see Fig. 2.

Finally we illustrate the theory by including the influence of the ground impedance on the insertion loss for some outdoor surfaces, as well as for the model surface, the felt. In connection with the theory of a porous layer⁸ the impedances of some outdoor surfaces were measured. The resulting insertion loss responses for a given relatively high screen are shown in Fig. 16. Considering the different impedances [cf. Ref. 8, Fig. 10] or ground attenuation at low receiver heights, for which the impedance is estimated [cf. Ref. 8, Fig. 9], the insertion loss responses do not differ much at a higher position of the receiver. In Fig. 16 there is also a comparison with the felt (in full scale) from the model experiment in Sec. VI. Although the felt was not chosen to scale the outdoor surfaces [the ground attenuation responses differ, especially in the range 200–500 Hz (full scale frequencies), cf. Ref. 8, Figs. 6 and 9 at 2–5 kHz] this felt has an insertion loss response very similar to the outdoor grounds and is in this respect much better than the

response for the hard ground. It is likely that the method of measuring the ground impedance is sensitive enough for the selection of scale model grounds as to the insertion loss.

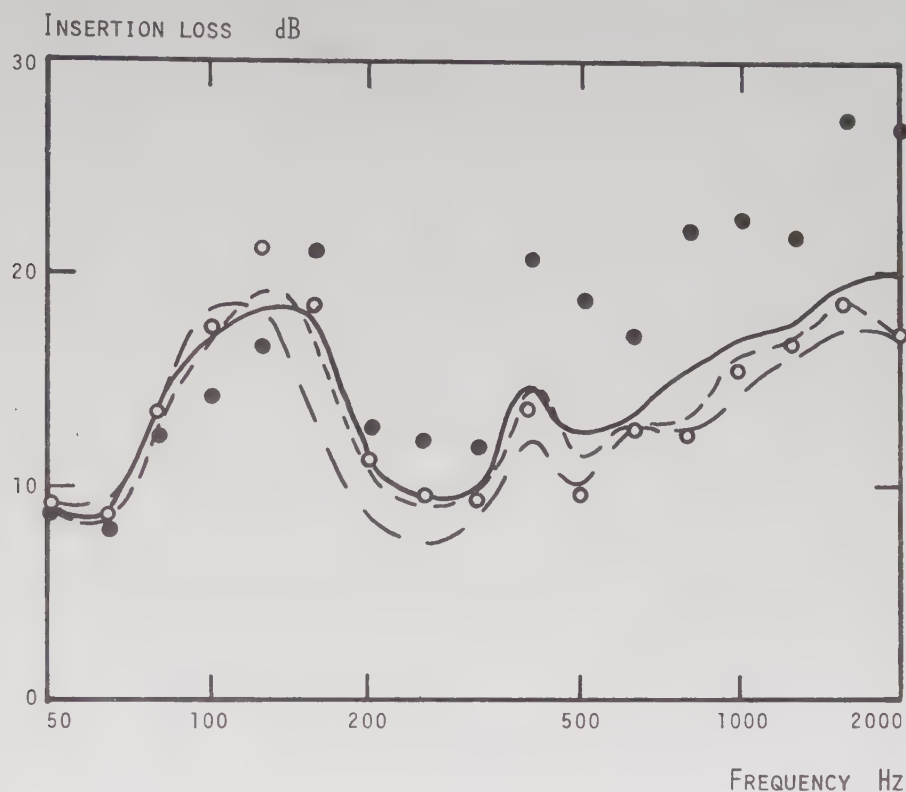


FIGURE 16. Calculated insertion loss responses for some outdoor ground surfaces (grassland, stubble field and newsown) and a scale model surface (the felt from Sec. VI, here in full scale). Calculations according to Sec. V.

Solid line, grassland: $a = 0.25$, $b = 10^\circ$, $c = 7500$ Hz and $d = 425$ Hz; large-dash line, stubble field: $a = 0.6$, $b = 10^\circ$, $c = 12000$ Hz and $d = 500$ Hz; small-dash line, newsown: $a = 0.6$, $b = 10^\circ$, $c = 30000$ Hz and $d = 1000$ Hz. Open-circle line, felt: $a = 0.51$, $b = 0^\circ$, $c = 19000$ Hz and $d = 870$ Hz; filled-circle line, hard ground: $v = 0$, see Eq. (10).

Geometry: $(x_S, y_S, z_S) = (10, 0, 0.1)$ m; $(x_R, y_R, z_R) = (-10, 0, 2.0)$ m; $(Y_0, Y_1, Z_1) = (-12.5, 12.5, 3.0)$ m, see Fig. 2.

VIII. SOME COMMENTS

This report has been preceded by two papers within the same field, and it is the final contribution to this series. They are all interconnected, and it might be useful to give a brief review of them. For more detailed information about the background the reader is referred to each introduction.

The first paper, Ref. 7, paid attention to one important detail: What is the origin of the difference between Ingard's and Wenzel's asymptotical solutions to the problem of impedance boundary with a given impedance? In a special case Wenzel's solution contains a specific term, the surface wave term, which is missing in Ingard's solution. This is confusing as the solution is unique, and curiously enough there is a historical parallel at the beginning of the century in the electromagnetic field (for a corresponding problem a solution by Weyl, similar to Ingard's, does not contain a surface wave term as does the one by Sommerfeld). The paper shows how Ingard's solution in some special cases could be revised (Ref. 7, Secs. III and IV) to agree asymptotically with Wenzel's (Ref. 7, Secs. VI and VII). This is not only of theoretical importance. Ingard's solution is of great importance, being the most numerically efficient exact solution to the problem. But the paper does not give any answer to the question of whether the surface wave term has any physical significance at all. The experimental verification of it, if possible, still remained to be done.

To do so we have to revert to the physical mechanism behind the point impedance. This was done in the second paper, Ref. 8. It had been a general suggestion that outdoor surfaces, such as grass, consisted of some kind of layer or layers, and also that the surface was locally reacting, but these two approaches were never unified. The model — somewhat arbitrary chosen — was a single porous layer on a hard backing. It was shown (Ref. 8, Sec. III) that if the refraction index of the layer is sufficiently large, the layer reacts like a point impedance even for a point source. As by-products we may get knowledge of the frequency dependence of the point impedance (Ref. 8, Sec. IV), an approximate solution of high accuracy (Ref. 8, Sec. V) and a new method of measuring the point impedance (Ref. 8, Sec. VII). The experiments indicated that the suggested method of measuring the impedance of a

layer, e.g. grassland, is useful (Ref. 8, Secs. VIII and IX) but they also showed that the analysis of the first paper (Ref. 7) had not been in vain; the surface wave term could indeed dominate the field above a porous layer (a felt). Although there is much left to be done on this problem, especially outdoors, there seemed to be a possibility to extend this knowledge to another problem.

This has been done in the present report. By using the simple theory of physical optics, an approximate solution to the problem of a screen on an impedance boundary is developed (Sec. III). The solution of the first paper is, as mentioned above, numerically efficient, but it is not efficient enough to be used in this connection. This was the only reason to develop an approximate solution of the impedance boundary (Ref. 8, Sec. V). By applying these approximations to the solution of the problem when we have a screen too, one may get a still better solution, numerically seen (Sec. IV). The experiments in the present study (Secs. VI and VII) give credit both to the physical optics assumption and to the theory of a porous layer, especially the method of measuring the point impedance.

ACKNOWLEDGMENTS

The present study concludes a trilogy, the other two parts of which are "Reflection of waves from a point source by an impedance boundary" (Ref. 7) and "Sound propagation above a layer with a large refraction index" (Ref. 8). Together with "Theory and experiments on sound propagation above an impedance boundary" (Ref. 31) this series constitutes the author's thesis on sound propagation over plane grounds with and without screens. In this connection the author would like to express his appreciation to Prof. S.G. Lindblad for taking part in an inspiring dialogue, to Mr. B. Bengtsson and Mr. L. Persson for their valuable interest and help in the experimental part, and to Miss M. Johnsson for her patient aid in preparing and typing manuscripts at various stages of this work. The author also would like to express his gratitude to Prof. J. Peetre for his kind guidance with the first paper.

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